



M Robust Weighted Ridge Estimator in Linear Regression Model

Taiwo Stephen Fayose^{a,*}, Kayode Ayinde^b, Olatayo Olusegun Alabi^b

^aDepartment of Statistics, Federal Polytechnic Ado-Ekiti, Nigeria

^bDepartment of Statistics, Federal University of Technology, Akure, Nigeria

Abstract

Correlated regressors are a major threat to the performance of the conventional ordinary least squares (OLS) estimator. The ridge estimator provides more stable estimates in this circumstance. However, both OLS and Ridge estimators are sensitive to outlying observations. In previous studies, the robust ridge based on the M estimator suitably fit well to the model with multicollinearity and outliers in outcome variable. Since outlier is one of the sources of heteroscedasticity and the non – robust weighted least squares have been previously adopted to account for it. Therefore, this paper proposed and developed some novel estimators to handle three problems (multicollinearity, heteroscedasticity and outliers) simultaneously with the aim of identifying the most efficient (best) ones. The Ordinary Least Square (OLS) and M robust estimator in two weighted version (real weight (W_o) and one estimated weight (W_1)) were combined to respectively develop the M Robust Ridge and M Robust Weighted Ridge Estimators. Monte – Carlo simulation experiments were conducted on a linear regression model with three and six explanatory variables exhibiting different degrees of Multicollinearity, with heteroscedasticity structure of powers, magnitude of outlier in y direction and error variances and five levels of sample sizes. The Mean Square Error (MSE) was used as a criterion to evaluate the performances of the new and existing estimators. It is evident from the simulation results that the proposed estimators produced efficient estimates and hereby recommended.

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*Corresponding author tel. no: +234 8165264106

Email address: fayose_ts@fedpolyado.edu.ng (Taiwo Stephen Fayose)

1. Introduction

Linear regression model is a statistical tool that investigates the relationship between an outcome variable and one or more regressors [1]. The model is simply defined as follows:

$$ty_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_k x_{ik} + \varepsilon_i, i = 1, \dots, n, \quad (1)$$

where y_i is the outcome variable, $x_{i1}, \dots, x_{ik}$ are the regressors, $\beta_0, \beta_1, \dots, \beta_k$ are the unknown parameters to be estimated, ε_i denotes the disturbance term and it is assumed to be normally distributed with mean zero and constant variance σ^2 . The model is simply a simple regression model when there is one regressor. The parameters in model (1) are mostly estimated by the Method of Least Squares (MLS). MLS is generally preferred and possesses some attractive properties when the assumptions of the linear regression models are intact [2-3]. These include linear independency among the regressors; the disturbance terms must come from a normal distribution and has constant variance and others [4]. In practice most of the aforementioned assumptions are easily violated. For instance, literature has shown that linear dependency frequently exists among regressors which are termed multicollinearity [5]. There is tendency for perfect or strong or moderate linear dependency among the regressors [5-6]. The method of least squares is unbiased but inefficient when there is linear dependency among the regressors [7]. It yields regression coefficients whose absolute values are too large and whose signs may reverse with negligible changes in the data [8]. If the multicollinearity is not perfect but high, estimates of the regression coefficients are possible but they do have large standard errors [7]. The interpretation given to the regression coefficients may no longer be valid since the regression coefficients with large sampling error affects both inference and forecasting results from the model [41].

Several methods for handling multicollinearity include the ridge Regression [9], Liu estimator [10], Partial least square regression [11], Principal component regression [12], Liu – type estimator [13], Modified ridge – type estimator [14], Modified Liu – estimator [15], Kibria – Lukman estimator [16], Jackknife Kibria – Lukman estimator [17], and others. However, these estimators are not robust to outlier.

Outlier(s) are observation/observations that appears to differ from other observations under consideration [18]. The presence of outlier affects the performance of all the previously mentioned methods. It causes a noticeable change in model estimates such as predicted values, estimated variance, and others [19-21]. In addition, the assumption of normality is violated with the presence of outlying observation [19-20]. For instance, a single outlying point negatively influenced the performance of MLS because the breakdown point of the estimator is very low [19-21]. Robust Regression are the alternative methods to MLS when the model is contaminated with outliers or influential observations. The M-estimator is the most frequently used robust methods for dealing with outlier in the outcome variable [22]. Others include the MM – estimator, S – estimator, the Least Trimmed Squared estimator, Least Absolute Deviation estimator, Least Quantile of Squares estimator [23-27] etc.

Recent studies have shown that multicollinearity and outliers can jointly feature in a model. Efforts have been made to mitigate both problems by pooling together some of the existing methods for handling each of the problems individually [21, 28-33]. For instance, the robust ridge [32] was obtained by combining the M – estimator for handling outlier and the ridge estimator for accommodating multicollinearity. The robust Jackknife ridge estimator was derived to handle both problems [28].

Till date the weighted least squares have been adopted in the literature as a remedial measure for the problem of heteroscedasticity. Since outlier is an error problem and can be the reason for heteroscedasticity in a model [35]. We decided to propose a new technique for handling both multicollinearity and outliers by combining both the weighted ridge regression with some robust estimators. We compared the estimators' performance to that of some existing techniques. Section 2 contains the methodology. Section 3 presents the simulation design, while Section 4 presents simulation results and in appendix while Section 5 concludes with a conclusion.

2. Existing and Proposed Estimators

The matrix form of a linear regression models is defined as:

$$y = X\beta + U, \quad (2)$$

where $y_{(nx1)}$ is vector of dependent variable; $X_{(n \times p)}$ is matrix of regressors, $\beta_{(nx1)}$ is vector of the model parameters, and $U_{(nx1)}$ is vector of error terms/disturbance. The model in canonical form is given by:

$$y = Z\alpha + U, \quad (3)$$

where $Z = XQ$, $\alpha = Q'\beta$ and Q is the orthogonal matrix whose columns constitute the eigenvectors of $X'X$. Then, $Z'Z = Q'X'XQ = \Lambda = \text{diag}(\lambda_1, \dots, \lambda_p)$.

2.1. Some Alternative Ridge Estimators to OLSE

The ridge estimator [10] is defined as:

$$\hat{\beta}_{RIDGE} = (Z'Z + kI)^{-1}Z'y, \quad (4)$$

where k is the non-negative tuning parameter. Different means of deriving k exists in the literature. These include:

$$\hat{k}_i(HK) = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}, i = 1, 2, 3, \dots, p, \quad (5)$$

where $\hat{\sigma}^2 = \frac{\sum_{i=1}^n e_i^2}{n-p}$ is the mean square error from the MLS, α_i is the i^{th} element of the vector and is the regression coefficient from the MLS.

Following [36], k is given by:

$$KGRFA = \hat{k}_i^{Min}(FA) = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} \left\{ \left[\left(\frac{\hat{\alpha}_i^4 \lambda_{Min}^2}{4\hat{\sigma}^2} \right) + \left(\frac{6\hat{\alpha}_i^4 \lambda_{Min}}{\hat{\sigma}^2} \right) \right]^{\frac{1}{2}} - \left(\frac{\hat{\alpha}_i^2 \lambda_{Min}}{2\hat{\sigma}^2} \right) \right\}, \quad (6)$$

where $\lambda_{Min} = \text{Min}(\lambda_i) = 1, 2, 3, \dots, p$.

In this study, we also obtain the ridge parameter using the cross – validation approach [37-38].

2.2. Proposed Robust Weighted Ridge Estimator

Consequently, in this study, the proposed estimator called robust M weighted ridge estimator is defined as:

$$\hat{\alpha}_{KWT}(k, w, t) = (Z_k' \Omega_w^{-1} Z_k + k_t I)^{-1} (Z_k' \Omega_w^{-1} y_k), \quad (7)$$

where Z_k is canonical matrix associated ridge parameter k for handling multicollinearity. $\Omega_w^{-1} = P_w^{-1} P_w^{-1}$, $\Omega_w = P_w P_w$ measures the heteroscedastic structure of the error variance with weight w ; and k_t is the robust ridge parameter to handle Multicollinearity and Outlier jointly. The robust ridge parameters are defined based on the M – estimator. The M – Estimator chooses β to minimize rather than minimizing the sum of squared errors. The M – estimator is defined as

$$\sum_{i=1}^n \rho(y_i - x_i^T \hat{\beta}). \quad (8)$$

Thus, the robust ridge parameter is defined as follows:

$$KGRHK_M = \frac{\Omega_M}{\hat{\alpha}_{i,M}^2}, \quad (9)$$

where Ω_M is the variance for the M – estimate, $\hat{\alpha}_{i,M}^2$ is the M – estimate.

$$KGRFA_M = \hat{k}_i^{Min}(FA) = \frac{\Omega_M}{\hat{\alpha}_i^2} \left\{ \left[\left(\frac{\hat{\alpha}_i^4 \lambda_{Min}^2}{4\Omega_M} \right) + \left(\frac{6\hat{\alpha}_i^4 \lambda_{Min}}{\Omega_M} \right) \right]^{\frac{1}{2}} - \left(\frac{\hat{\alpha}_i^2 \lambda_{Min}}{2\Omega_M} \right) \right\}. \quad (10)$$

As earlier mentioned, we adopted the weighted method defined in equation (7) to handle multicollinearity, heteroscedasticity and outlier (y direction) in this research. Since the weight Ω_w is not always known in practice, we therefore to compute the weight. The Weighted least squares (WLS) procedure is as follows:

1. Obtain the residual square U_i^2 of the estimation.
2. Formulate an auxiliary regression to obtain an estimate of the error variance. This is done by regressing the log of the squared residuals from the OLS on the original regressors and their squares. The log transformation is performed to ensure that the estimated variances are all non – negative:

$$\log U_i^2 = f(x_1, x_2, \dots, x_p, x_1^2, \dots, x_p^2). \quad (11)$$

3. Obtain the error variance by estimating the fitted values as:

$$\log \widehat{U_i^2} = T^*. \quad (12)$$

4. The fitted values from the regression are called T^* . The weight series for the final WLS is then formed as:

$$W = \sqrt{\frac{1}{\exp(T^*)}}. \quad (13)$$

5. Using WLS to estimate the parameters, where the reciprocal of the estimated variance is used as weight to correct for the presence of heteroscedasticity in the model. The initial weight that has been adopted in previous study is called W_0 . We suggested a new weight W_1 which is defined as follows:

$$W_0 = f(E(y_i)^{2\delta}). \quad (14)$$

$$W_1 = f(x_1, x_2, \dots, x_p, x_1^2, x_2^2, \dots, x_p^2), \quad (15)$$

where p is the number of regressors.

The proposed estimator can return to Generalized Least Squares (GLS), Robust Ridge and OLS estimators in the following cases:

$$\hat{\beta}_{RWR}(0, \Omega, 0) = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y = \hat{\beta}_{GLS}, \text{ the GLS estimator.} \quad (16)$$

$$\hat{\beta}_{RWR}(k_R, 1, 0) = (X'X + k_R I)^{-1}X'y = \hat{\beta}_{RR}, \text{ the Robust Ridge estimator.} \quad (17)$$

$$\hat{\beta}_{RWR}(k, \Omega, 0) = (X'\Omega^{-1}X + kI)^{-1}(X'\Omega^{-1}y) = \hat{\beta}_{WRE}, \text{ the Weighted Ridge estimator.} \quad (18)$$

$$\hat{\beta}_{RWR}(0, 1, 0) = (X'X)^{-1}X'y = \hat{\beta}, \text{ the OLS estimator.} \quad (19)$$

3. Simulation Procedure and Design

A Monte Carlo simulation study is performed in the study to show the performance of the proposed estimator over some existing estimators in literature.

Consider the linear regression of the form:

$$y_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \dots + \beta_p X_{tp} + U_t, \quad (20)$$

where $t = 1, 2, \dots, n$; $p = 3, 6$, $U_t \approx N(0, \sigma^2 I_n)$, $X_{ti}, t = 1, 2, \dots, n$; $i = 1, 2, \dots, p$ are fixed regressors. The regressors are generated using the following procedure [36]:

$$X_{ti} = (1 - \rho^2)^{\frac{1}{2}} Z_{ti} + \rho Z_{t,p}, \quad (21)$$

where Z_{it} is independent standard normal distribution with mean zero and unit variance, ρ is the correlation between any two regressors and p is the number of exposure variables. The error terms U_t were generated to be normally distributed with mean zero and variance σ_i^2 . The heteroscedasticity problem was introduced into the model with equation

$$u_i = z\sigma \sqrt{E(y_i)^{2\delta}}. \quad (22)$$

U_i is the error term and δ_i^2 is the heteroscedasticity variance considered. The study used Monte Carlo simulation to conduct the experiment with varying parameters such as sample sizes ($n = 15, 20, 30, 50, 100$); level of multicollinearity ($\rho = 0, 0.8, 0.9, 0.95, 0.99, 0.999$) and delta ($\delta = 1, 2$). In the study, σ^2 values were 1, 25 and 100. $E(y_i)$ = expected value of the regression under consideration. $y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u_i$ for $p = 3$ and $y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + u_i$ for $p = 6$. Outliers were introduced into the y direction.

1. A specific percentage of outliers (10%) of the generated data is randomly sampled using Equal Probability Selection Method (EPSM). It was then inflated with outliers according to step 3.
2. Select the degree of outlier (k) which is the outlier magnitude. In the study, $k = 1, 5$.
3. Let $y_1, y_2, \dots, y_n$ be a random sample of a normally distributed random variable Y respectively. Then,

$$y_i^* = k[\max(y)] + y_i$$

where y_i^* = inflated observation replaces y_i

y_i = sampled observation to be replaced.

k = degree of an outlier.

For more detail on the generating the outliers [see 21, 40]. The experiment was repeated 1000 times (number of replication). The performances of the estimators were compared using the Mean Square Error criterion. For any estimator $\hat{\beta}$, MSE is defined as follows:

$$MSE(\hat{\beta}) = \frac{1}{1000} \sum_{i=1}^p \sum_{j=1}^{1000} (\hat{\beta}_{ij} - \beta_i)^2, \quad (23)$$

where $\hat{\beta}_{ij}$ is i^{th} element of the estimator $\hat{\beta}$ in the j^{th} replication which gives the estimate of β_i . β_i are the true value of the parameter previously mentioned. Estimator with the minimum MSE was considered best. We compare the following:

1. OLS.
2. Robust ridge using $KGRFA_M$ as the ridge parameter. We abbreviate this as MFA.
3. Robust ridge using $KGRHK_M$ as the ridge parameter. We abbreviate this as MHK.
4. Robust ridge using CV as the ridge parameter. We abbreviate this as MCV.
5. Robust weighted ridge using $KGRFA_M$ as the ridge parameter and adopting the initial weight. We abbreviate this as MWOFA.
6. Robust weighted ridge using $KGRHK_M$ as the ridge parameter and adopting the initial weight. We abbreviate this as MWOHK.
7. Robust weighted ridge using CV as the ridge parameter and adopting the initial weight. We abbreviate this as MWOCV.
8. Robust weighted ridge using $KGRFA_M$ as the ridge parameter and adopting the new weight. We abbreviate this as MW1FA.

9. Robust weighted ridge using $KGRHK_M$ as the ridge parameter and adopting the new weight. We abbreviate this as MW1HK.
10. Robust weighted ridge using CV as the ridge parameter and adopting the new weight. We abbreviate this as MW1CV.

The MSE obtained by each estimator was ranked for each degree of multicollinearity, heteroscedasticity, outliers, and error variance. The degrees of multicollinearity, heteroscedasticity, outliers, and error variance were tallied to determine the number of times each estimator had the smallest MSE (rank 1 and 5). An estimator is optimal if it has the most counts, the mode.

4. Results and Discussion

We represented the outcomes visually and in Table 1.

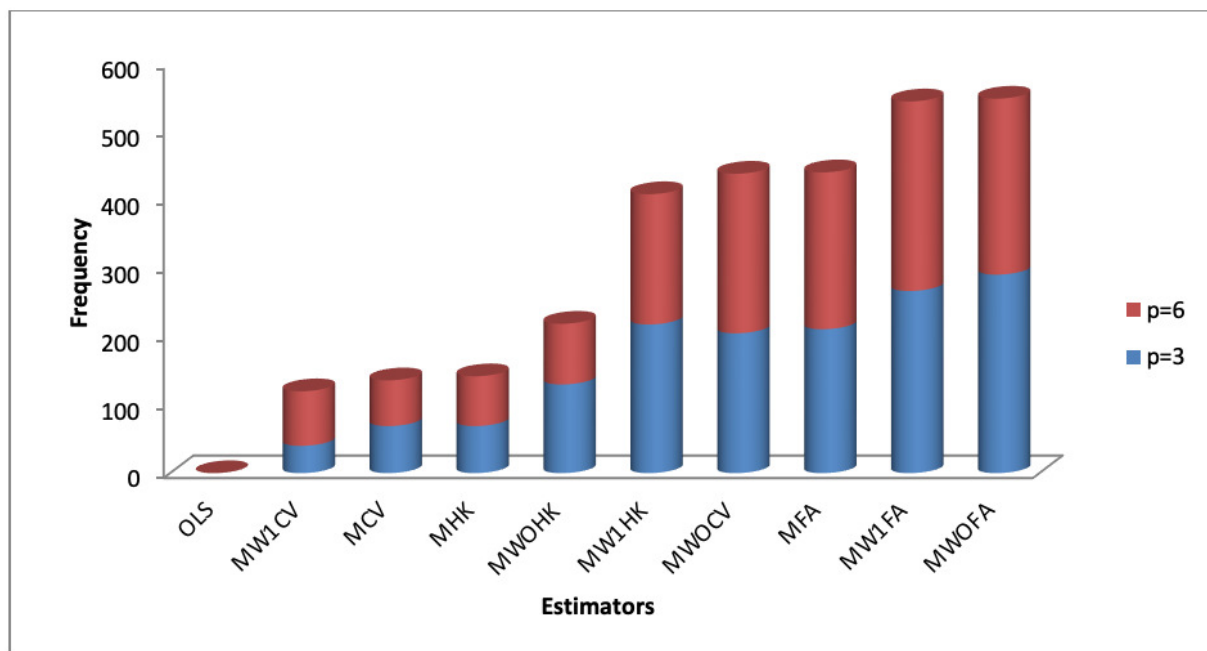


Figure 1. Component Bar Chart showing frequency of counts at which MSE is minimum ranking between 1 and 5 at $p = 3$ and $p = 6$ for M Estimation Methods

Figure 1 shows that the MWOFA is the best (most efficient) estimator when dealing with the model's multicollinearity, heteroscedasticity, and outlier problems. i.e., proposed Robust M Weighted Least Squares with Real Weight and One parameter ridge estimator of Fayose and Ayinde [36] followed by MW1FA i.e. proposed Robust M Weighted Least Squares with Weight One and One parameter ridge estimator of Fayose and Ayinde [36] and MFA proposed Robust M and One parameter ridge estimator of Fayose and Ayinde [36] is the 3rd most efficient estimator to handle the three problems simultaneously.

From Figure 2, MW1FA dominates across all sample sizes except at sample size 100 when $p = 3$ but performed across all sample sizes at $p = 6$ respectively. Similarly, MWOFA performed across all sample sizes at $p = 3$ but at $p = 6$, the estimator performance improved as sample size increase from 15 to 100 respectively. Finally, OLS estimator didn't perform at all sample sizes from Figure 2.

The simulation results are available in the appendix (Table A. 1 to A. 60) but for ease of comparison the results are summarized in Table 1.

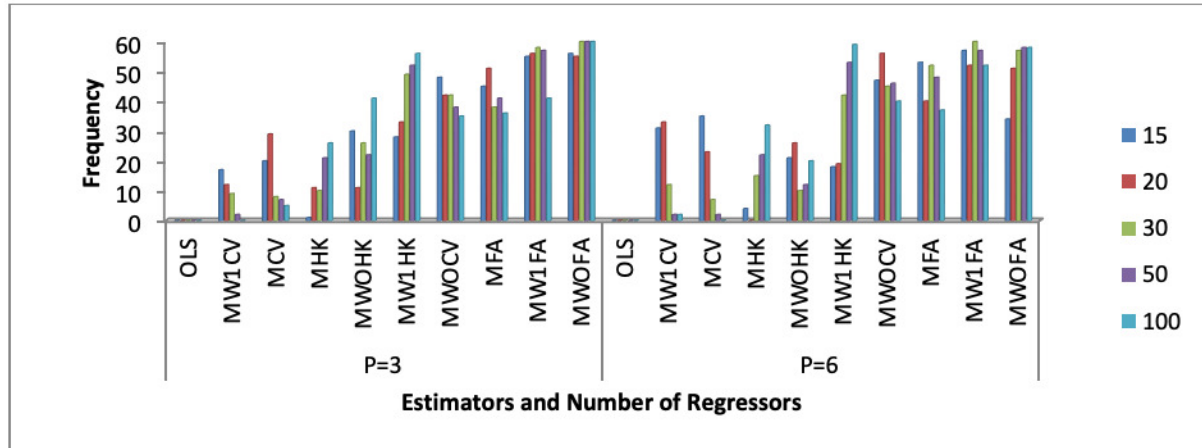


Figure 2. Multiple Bar chart showing performance of the estimators at different sample sizes when $p = 3$ and $p = 6$

Table 1. Number of Times Each Estimator Produced Minimum MSE when counted over the levels of Multicollinearity, Heteroscedasticity, Outlier in (y direction) and Error Variance.

p	Estimators	Sample Size (n)					TOTAL	RANK
		15	20	30	50	100		
3	OLS	0	0	0	0	0	0	10 th
	MW1CV	17	12	9	2	0	40	9 th
	MCV	20	29	8	7	5	69	7 th
	MHK	1	11	10	21	26	69	7 th
	MWOHK	30	11	26	22	41	130	6 th
	MW1HK	28	33	49	52	56	218	3 rd
	MWOCV	48	42	42	38	35	205	5 th
	MFA	45	51	38	41	36	211	4 th
	MW1FA	55	56	58	57	41	267	2 nd
	MWOFA	56	55	60	60	60	291	1 st
6	OLS	0	0	0	0	0	0	10 th
	MW1CV	31	33	12	2	2	80	7 th
	MCV	35	23	7	2	0	67	9 th
	MHK	4	0	15	22	32	73	8 th
	MWOHK	21	26	10	12	20	89	6 th
	MW1HK	18	19	42	53	59	191	5 th
	MWOCV	47	56	45	46	40	234	3 rd
	MFA	53	40	52	48	37	230	4 th
	MW1FA	57	52	60	57	52	278	1 st
	MWOFA	34	51	57	58	58	258	2 nd

NOTE: Estimator with highest frequency at each sample size is bolded at $p = 3$ and $p = 6$.

From Table 1, when $p = 3$, it can be observed that the most efficient estimator is the proposed MWOFA i.e. proposed Robust M Weighted Least Squares with Real Weight and One parameter ridge estimator of Fayose and Ayinde [36] as observed in Figure 1. MW1FA i.e. proposed Robust M Weighted Least Squares with Weight One and One parameter ridge estimator of Fayose and Ayinde [36] is ranked second while MW1HK i.e. proposed Robust M Weighted Least Squares with Weight One and One parameter ridge estimator of Hoerl and Kennard [10] is the third most efficient estimators to handle the three problems simultaneously but when $p = 6$, it can be observed that the most

efficient estimator is proposed MWIFA i.e. proposed Robust M Weighted Least Squares with Weight One and One parameter ridge estimator of Fayose and Ayinde [36] but proposed MWOFA i.e. proposed Robust M Weighted Least Squares with Real Weight and One parameter ridge estimator of Fayose and Ayinde [36] ranked second while proposed MWOCV i.e. proposed Robust M Weighted Least Squares with Real Weight and k parameter from Cross Validation came third. As expected, the ordinary least square estimator is sensitive to multicollinearity, heteroscedasticity and outliers.

5. Conclusion

In order to forecast the response variable from a set of regressors, linear regression models (LRMs) are frequently utilized. Multicollinearity is known to decrease the performance of the conventional least squares technique. The ridge estimator and other estimators offer superior estimates in this situation. Most of these techniques, including OLS, Ridge, and others, are, however, vulnerable to outliers. Robust estimators, such as the M – estimator, offer more reliable predictions when there is an outlier in LRMs. The heteroscedasticity problem in LRMs is also solved by weighted least square estimators. The three problems may truly coexist in models in practice. To take them into account, the well – known robust weighted ridge estimator is used. For the goal of addressing the three problems, we deployed the reliable proposed robust M weighted ridge estimator in this work and provided a new weight. We performed a simulation analysis and found that the recommended approaches suit the model better than the ones already in use. Though the Robust M ridge estimators compete favorably with the Robust M weighted ridge estimators in few cases and hereby recommended.

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Appendix A.

Table A. 1. MSE result when the magnitude of outlier is 1, $\rho = 0.8$ and $\sigma^2 = 1$.

Table A.1: MSE result when the magnitude of outlier is 1, $\rho = 0.8$ and $\sigma = 1$.										
	$\sigma^2=1$			$\delta^2 = 1$			$k = 1$			
$\rho = 0.8$										
Estimators	15	20	p=3	50	100	15	20	p=6	50	100
OLS	4.50169	6.458829	1.201181	1.890803	1.081163	6.007301	5.819461	3.811372	2.557778	1.7415706
MHK	4.195663	4.218969	2.789803	3.733219	3.969503	2.070789	2.443592	1.372089	0.854827	1.7447081
MFA	3.837991	4.151111	3.092445	4.057577	3.817471	1.937425	3.166128	1.680581	1.235198	2.2724307
MCV	5.458111	6.71866	3.949961	5.983232	5.08614	2.92744	4.445921	2.439645	2.384762	3.5976345
MWOHK	2.052626	0.997998	0.9335	1.988887	0.427857	2.149939	1.465209	0.865517	1.584959	2.7822369
MWOFA	2.562012	0.736243	1.098923	1.640477	0.342957	2.312756	1.327496	0.731752	1.791714	2.2568234
MWOCV	2.974054	1.869652	1.106521	2.132809	0.630302	2.189589	1.332481	1.015962	1.372888	2.3828109
MW1HK	1.660964	2.137636	1.021394	1.762976	1.122655	1.569354	1.907788	1.246291	1.153373	1.1515202
MW1FA	1.646703	1.878453	1.031555	1.843594	1.106383	1.376604	1.822924	1.315747	1.211407	1.2224996
MW1CV	2.597731	3.580109	1.277628	2.786461	1.507647	1.770436	1.856916	1.79975	1.560065	1.5568059

NOTE: k = Magnitude of outlier, ρ = multicollinearity level, σ^2 = error variance value and δ^2 = delta

Table A. 2. MSE result when the magnitude of outlier is 5, $\rho = 0.8$ and $\sigma^2 = 1$.

		$\sigma^2=1$	$\delta^2 = 1$	$k= 5$							
$\rho = 0.8$											
Estimators	15	20	$p=3$ 30	50	100	15	20	$p=6$ 30	50	100	
OLS	52.00884	98.36983	21.3197	35.04379	22.60642	64.33209	46.10931	59.37381	49.62696	36.366823	
MHK	3.105865	3.104857	2.182497	2.64798	2.92819	1.742191	2.263872	0.894581	0.681373	1.2135444	
MFA	4.579363	5.917063	4.300623	4.711872	4.552968	2.525205	3.974235	2.316761	1.621457	3.5581697	
MCV	6.584768	17.13538	10.6821	15.5406	15.17763	2.873974	1.343104	7.112252	10.25932	17.062852	
MWOHK	5.506845	3.244525	0.894383	1.90554	0.842915	2.592359	1.238435	2.843836	1.900405	2.0827081	
MWOFA	2.603143	1.634292	0.856246	1.637579	0.94867	3.590838	1.910448	1.6358	1.827237	2.1888284	
MWOCV	3.217124	5.161491	1.97008	2.681985	1.387907	1.841295	2.082102	1.850391	1.624438	2.859448	
MW1HK	1.311164	1.505674	0.901974	1.479722	1.297096	1.165998	1.111067	1.173248	1.147643	1.3062733	
MW1FA	1.4425	1.962677	1.144633	2.053798	1.412134	1.15225	0.937132	1.408899	1.400734	1.5432293	
MW1CV	3.40092	7.339942	2.075093	8.292388	6.746614	1.753815	1.671528	2.920434	3.927021	6.1400766	

Table A. 3. MSE result when the magnitude of outlier is 1, $\rho = 0.8$ and $\sigma^2=1$.

		$\sigma^2=1$	$\delta^2=2$	$k=1$							
$\rho=0.8$											
Estimators	15	20	$p=3$ 30	50	100	15	20	$p=6$ 30	50	100	
OLS	22.24595	58.26637	3.791314	12.39324	6.63006	22.94177	28.51651	22.07614	17.03481	10.043072	
MHK	7.258841	10.38776	2.988486	4.006786	3.770397	4.36169	7.332954	1.91606	1.01624	1.6593004	
MFA	4.230421	5.945045	3.355839	4.228553	3.966994	2.513137	5.343266	2.413234	1.562605	2.7785995	
MCV	7.028709	18.41357	5.0172	8.804335	8.28888	5.083089	16.82231	3.925495	4.993774	7.4238141	
MWOHK	1.503681	2.41336	0.878834	1.122405	1.100227	1.42318	0.899862	1.102297	1.267049	1.1042173	
MWOFA	1.327055	1.185302	0.938854	1.079607	1.026431	1.108447	0.990073	0.968123	1.078289	1.05026	
MWOCV	1.686774	2.214379	0.945656	1.161342	1.076512	2.406711	1.075815	0.988795	1.01781	1.0438579	
MW1HK	2.433025	3.878158	1.003994	1.197943	1.045065	3.093867	4.402331	1.375888	1.091355	1.1834061	
MW1FA	2.06661	2.362149	1.042571	1.603632	1.056758	1.775524	2.636245	1.513506	1.275044	1.2934976	
MW1CV	6.497353	39.48256	1.733428	5.905638	2.763281	3.769533	19.25572	3.882983	2.900698	2.7664956	

Table A. 4. MSE result when the magnitude of outlier is 5, $\rho = 0.8$ and $\sigma^2=1$.

		$\sigma^2=1$	$\delta^2 = 2$	$k = 5$							
$\rho = 0.8$											
Estimators			$p=3$					$p=6$			
	15	20	30	50	100	15	20	30	50	100	
OLS	233.5211	664.5821	66.64768	212.9523	135.458	186.4178	167.1862	299.8078	308.0865	203.70151	
MHK	6.511227	7.096691	2.200595	2.44563	2.002992	3.993729	9.705513	1.209477	0.791128	1.1034944	
MFA	4.887598	6.901374	4.459924	4.901805	4.804976	3.11691	7.474737	3.253774	2.158391	4.3246104	
MCV	7.07437	38.76836	15.72515	38.91696	39.41315	4.639084	2.544836	18.49937	37.55287	48.575156	
MWOHK	6.015725	22.53331	0.958152	2.27946	1.324861	3.817568	0.986717	4.283382	1.955808	1.4480311	
MWOFA	1.547742	1.611797	0.931939	1.156433	0.998479	3.347935	1.409977	1.109475	1.094012	1.1006818	
MWOCV	2.493408	3.767267	0.937806	1.529654	1.209514	5.692878	1.524256	1.105512	1.064266	1.1258234	
MW1HK	2.163703	1.787094	0.982017	1.045944	1.027025	2.133111	1.310584	1.477443	1.077265	1.1746695	
MW1FA	1.821076	2.618232	1.300325	2.251899	1.632228	1.38683	2.930959	1.906311	1.656071	1.7650437	
MW1CV	11.68363	88.42392	4.044922	29.84401	21.84954	3.57931	11.63067	6.364818	11.14041	17.256431	

Table A. 5. MSE result when the magnitude of outlier is 1, $\rho = 0.8$ and $\sigma^2 = 25$.

	$\sigma^2 = 25$		$\delta^2 = 1$	k = 1						
$\rho = 0.8$			p=3			p=6				
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	87.0185	106.9423	19.77245	30.25489	14.80351	118.3384	127.1504	64.80306	39.35466	26.140892
MHK	34.54718	28.87212	6.631798	6.224357	6.053313	19.5297	17.35488	9.623174	4.394518	4.8201089
MFA	5.04482	6.726973	3.83575	4.105359	3.917837	4.309234	11.24251	3.641402	1.911559	3.1369158
MCV	23.2045	27.19247	8.344776	5.345351	10.5478	13.98962	19.78954	9.563576	8.507662	11.65613
MWOHK	2.354591	14.49666	3.379161	4.156625	5.303307	17.89792	1.035111	7.805602	5.139348	3.6623557
MWOFA	2.904184	2.644286	0.766805	2.498637	2.595849	14.25879	1.556397	3.577212	2.751869	5.1067686
MWOCV	37.10108	45.7128	5.310909	5.677862	16.59014	14.53687	1.017086	8.461908	3.754592	13.478032
MW1HK	7.573605	10.52893	3.106761	2.666601	2.303211	19.9204	62.347	6.640777	2.875555	2.4408067
MW1FA	4.979677	6.757039	2.246632	3.071388	3.696122	5.418459	5.924721	4.397254	3.251046	4.2176947
MW1CV	90.88197	106.3738	23.39892	25.77535	54.30798	31.25212	12.05652	48.84245	46.11342	50.633179

Table A. 6. MSE result when the magnitude of outlier is 5, $\rho = 0.8$ and $\sigma^2 = 25$.

		$\sigma^2 = 25$	$\delta^2 = 1$	$k = 5$						
$\rho = 0.8$										
Estimators	15	20	p=3 30	50	100	15	20	p=6 30	50	100
OLS	944.6	1033.357	273.8835	425.2101	275.866	766.0531	618.4721	758.0026	621.3506	469.48631
MHK	80.30875	24.25953	6.223256	8.522935	4.415682	25.84917	18.1716	8.362899	3.896483	3.2483387
MFA	7.073454	7.642053	4.888627	5.210384	4.958893	4.778895	7.805213	4.75453	2.585144	4.8268876
MCV	6.961269	64.95805	36.1811	75.77154	64.96066	10.64175	39.4779	45.4221	60.18337	85.439041
MWOHK	121.0286	219.2783	44.96095	62.24522	48.13796	75.09828	1.213591	106.8289	49.83505	13.564983
MWOFA	3.807785	4.426391	2.12356	5.944258	6.263661	29.50652	2.797201	11.86196	10.21358	11.605538
MWOCV	30.56676	106.0102	48.16137	42.27862	34.81286	4.523781	37.02435	44.83943	10.36269	53.616981
MW1HK	9.110192	2.582651	1.311264	1.778211	1.07068	9.838717	11.70667	4.221803	1.499692	1.1981365
MW1FA	3.325757	4.04325	2.473613	4.455221	5.292281	2.938388	2.876901	4.036318	3.616341	5.4109358
MW1CV	237.9224	182.0523	70.77511	301.7904	358.7768	21.66141	74.12963	77.32603	149.8075	272.71586

Table A. 7. MSE result when the magnitude of outlier is 1, $\rho = 0.8$ and $\sigma^2=25$.

		$\sigma^2 = 25$	$\delta^2 = 2$	$k = 1$						
$\rho = 0.8$										
Estimators			p=3					p=6		
	15	20	30	50	100	15	20	30	50	100
OLS	746.8849	1339.619	76.05578	233.8772	138.3861	516.7328	729.6907	487.4434	370.7037	208.08249
MHK	256.8898	128.4885	11.48965	25.47828	7.450378	77.38702	131.803	29.57431	9.10952	6.2972864
MFA	10.39086	13.40278	4.348638	5.220044	4.313148	6.648208	19.04809	7.130578	3.435037	4.2791248
MCV	57.07362	91.45073	17.09517	39.64198	35.48694	31.97385	149.9877	28.06579	36.6473	45.611659
MWOHK	6.237732	136.2336	2.210747	12.21814	4.635188	10.5257	1.011834	20.68363	7.288659	6.5899431
MWOFA	1.369787	3.857002	0.867975	1.646589	1.078522	9.446523	1.561435	1.894475	1.580019	2.0583031
MWOCV	42.0544	27.76991	1.110643	3.811551	2.656662	6.649994	2.228665	2.458312	1.245398	1.9929993
MW1HK	28.42742	24.60018	2.133679	2.384961	1.011577	82.41204	55.57108	7.398861	1.401936	1.1568664
MW1FA	9.592737	10.93189	2.101966	3.177666	3.799593	9.447525	34.47947	4.879533	2.834919	3.8653821
MW1CV	533.3249	304.2044	59.63495	410.6352	178.4614	69.16496	669.5382	162.889	154.3596	146.20196

Table A. 8. MSE result when the magnitude of outlier is 5, $\rho = 0.8$ and $\sigma^2=25$.

		$\sigma^2 = 25$	$\delta^2 = 2$	$k = 5$						
$\rho = 0.8$										
Estimators	15	20	p=3 30	50	100	15	20	p=6 30	50	100
OLS	4079.272	15151.6	1157.383	4076.122	2918.119	3181.833	2711.559	6057.684	6328.32	4000.9242
MHK	143.9501	130.7544	9.103765	14.2455	3.244545	88.95522	140.8161	22.12911	4.406916	2.5314486
MFA	7.470816	8.288423	5.006952	5.255736	5.176981	6.309414	12.90188	6.90875	3.407658	5.4844677
MCV	3.857487	241.2946	82.38966	272.2785	340.038	31.13068	277.6659	160.5423	293.9719	317.56141
MWOHK	343.3053	3724.361	35.07922	310.1391	69.09123	199.0418	0.997734	483.3772	155.4427	134.05118
MWOFA	2.76981	6.190277	0.838089	4.589014	2.098415	30.89585	2.328075	5.185825	3.070667	4.5196411
MWOCV	15.19304	53.26776	4.522573	19.57892	23.99118	10.02656	7.15808	13.69893	5.43063	6.8087709
MW1HK	2.96654	26.12807	1.549563	1.174201	0.966382	34.15903	26.29915	13.37979	2.020285	1.0255083
MW1FA	5.247486	5.132098	2.923481	4.136887	5.653894	4.594849	6.217013	5.040665	3.825783	5.7548026
MW1CV	559.0372	2407.098	91.98125	844.0348	1170.86	50.23107	121.0554	160.4995	518.5283	784.18028

Table A. 9. MSE result when the magnitude of outlier is 1, $\rho = 0.8$ and $\sigma^2=100$.

	$\sigma^2 = 100$		$\delta^2 = 1$	$k = 1$						
$\rho = 0.8$			$p=3$	$p=6$						
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	409.7383	369.1567	73.73116	114.5864	55.22656	460.3995	503.7327	253.2391	151.2933	98.882335
MHK	184.7339	93.4618	17.87804	19.05982	12.81007	79.02541	54.79665	37.14492	16.13412	14.168618
MFA	7.135436	9.224764	4.224916	4.163501	4.186275	6.56666	18.04239	5.745134	2.688284	3.7857576
MCV	62.34637	61.79533	17.0162	7.741529	23.63038	31.0599	29.03414	24.99152	20.76557	25.709397
MWOHK	2.096125	17.6003	9.888358	5.956682	17.88755	29.39168	1.164001	24.7995	12.34571	7.5127784
MWOFA	3.066921	3.075139	1.4549	4.798629	5.050915	24.38006	1.725922	8.038992	7.013256	8.8162578
MWOCV	116.55	182.576	30.87674	28.99869	56.81607	36.53866	1.074005	30.78769	8.837895	44.999431
MW1HK	17.55768	34.2202	7.209946	2.868435	3.259756	89.9337	185.2837	17.04556	4.795323	2.7988464
MW1FA	8.255017	9.128896	2.913673	2.571664	4.514374	11.98016	10.57767	8.278189	4.805221	5.5244505
MW1CV	487.782	310.6491	123.9433	122.7816	384.3418	102.0765	8.827365	285.4215	288.7703	276.29693

Table A. 10. MSE result when the magnitude of outlier is 5, $\rho = 0.8$ and $\sigma^2=100$.

		$\sigma^2 = 100$	$\delta^2 = 1$	$k = 5$						
$\rho = 0.8$										
Estimators	15	20	p=3 30	50	100	15	20	p=6 30	50	100
OLS	3430.129	4637.822	1002.401	1739.427	1045.632	2833.318	2524.786	2901.678	2332.625	1732.9088
MHK	310.2301	94.17698	17.57644	30.87898	10.4588	102.6268	63.81744	35.55362	14.56274	10.028995
MFA	8.111069	8.252448	5.239883	6.415041	5.100694	6.436165	10.38693	6.527452	3.158647	5.3516238
MCV	10.37901	113.8458	65.65754	71.49613	180.224	22.44923	114.9496	116.9294	149.2052	199.44493
MWOHK	166.233	914.7312	199.0613	127.3694	245.0201	362.2256	1.240674	520.8752	191.7432	80.316211
MWOFA	3.941718	4.587115	5.894572	13.40749	14.15958	48.40707	3.526439	26.62753	28.05969	22.995692
MWOCV	96.33039	351.471	36.64097	334.5972	127.3168	12.73141	4.548457	137.8236	31.53406	319.68974
MW1HK	16.93389	3.702009	1.416371	1.312697	1.11019	27.7807	49.6088	13.81262	1.639792	1.1760334
MW1FA	5.825057	4.436324	2.667291	3.820808	6.044314	5.719683	6.635526	5.944215	4.374398	6.4422544
MW1CV	1001.583	623.4131	229.5749	58.13114	1395.259	81.17993	330.8673	333.8754	617.2026	1108.9625

Table A. 11. MSE result when the magnitude of outlier is 1, $\rho = 0.8$ and $\sigma^2=100$.

		$\sigma^2 = 100$	$\delta^2 = 2$	$k = 1$						
$\rho = 0.8$										
Estimators	15	20	$p=3$	50	100	15	20	$p=6$	50	100
OLS	1928.163	4981.912	284.3038	881.7713	547.3737	1964.663	2932.92	1926.075	1461.751	814.36743
MHK	381.0092	662.9275	33.37124	76.69142	19.73743	350.7183	515.4049	120.8435	35.66399	21.212197
MFA	8.97203	15.50038	4.651872	5.449094	4.517285	14.50056	32.28203	11.50903	4.820477	5.1160264
MCV	71.77977	259.0038	32.67239	107.2413	93.1006	27.58826	342.5699	70.95267	93.18519	114.86162
MWOHK	73.30342	1606.189	9.409995	56.14628	16.86478	96.84593	1.031673	126.219	65.96999	51.392554
MWOFA	1.565826	4.208348	0.859598	2.958249	1.983201	36.78385	1.952047	3.645105	2.519198	4.0465073
MWOCV	67.56987	36.14492	3.414791	19.78193	12.39621	33.39068	2.349978	6.403585	2.805301	6.0055359
MW1HK	51.76955	104.4582	3.424295	2.395762	1.036867	526.9712	108.3483	20.46073	1.771616	1.1162043
MW1FA	7.059793	15.09694	2.450556	3.194114	4.407107	23.4157	49.54425	7.734183	3.4985	4.6644663
MW1CV	816.6348	1037.26	234.4776	1456.864	818.2534	137.3639	15514.4	673.8286	740.0779	715.51482

Table A. 12. MSE result when the magnitude of outlier is 5, $\rho = 0.8$ and $\sigma^2=100$.

		$\sigma^2 = 100$	$\delta^2 = 2$	$k = 5$						
$\rho = 0.8$										
Estimators	15	20	p=3	50	100	15	20	30	50	100
OLS	15797.64	43056.21	4090.158	12842.92	10950.06	12189.14	10239.38	23733.11	24793.94	15545.079
MHK	442.2767	402.8065	25.28519	58.92576	5.445267	357.7824	523.2084	95.20601	16.81604	7.4011868
MFA	6.57801	11.13505	5.186956	5.167242	5.176482	8.268004	15.98074	8.829164	3.856864	5.8032024
MCV	62.19363	9.593928	120.9038	546.2275	966.474	60.76888	636.956	362.9776	665.8452	674.50999
MWOHK	3691.315	243.1682	197.3618	1444.961	380.656	973.0642	1.014958	3163.397	1650.001	1269.3918
MWOFA	2.694646	7.253659	1.123116	11.57564	11.40001	103.1437	3.341172	11.93876	8.623405	13.820256
MWOCV	19.22689	762.0609	27.05127	25.66587	135.1062	14.1801	1.081285	42.30403	10.97708	29.629206
MW1HK	6.939763	312.7415	10.40212	1.036931	0.98283	148.5758	27.72697	36.09666	8.667863	1.0111412
MW1FA	3.585469	15.07708	3.324144	4.105898	6.532064	8.863516	10.82811	6.484463	4.310131	6.5012304
MW1CV	475.484	813.0363	345.5316	1261.43	3431.351	172.2956	365.1756	471.7346	2076.171	2839.9165

Table A. 13. MSE result when the magnitude of outlier is 1, $\rho = 0.9$ and $\sigma^2 = 1$.

		$\sigma^2=1$	$\delta^2 = 1$	$k= 1$						
$\rho = 0.9$										
Estimators	15	20	p=3	50	100	15	20	p=6	50	100
OLS	22.76808	16.21462	1.909198	3.427396	1.665241	11.20494	7.052194	8.831754	4.69497	3.006493
MHK	14.70576	6.857343	2.753329	3.686312	3.976132	2.701277	4.773478	1.75458	1.445234	1.759121
MFA	5.347869	4.668663	3.343847	4.401264	4.168414	2.834256	4.942916	1.892679	1.864469	2.404824
MCV	15.72066	9.76607	4.323775	6.855048	5.556427	3.964804	5.398695	3.47758	4.030465	4.414969
MWOHK	1.089223	4.214378	0.442106	1.912884	1.116885	1.988415	1.29901	2.158467	1.37874	3.339834
MWOFA	0.625462	2.62813	0.33002	2.055042	1.214538	2.456263	1.685106	2.673924	1.418537	2.067331
MWOCV	2.220797	4.626468	0.758834	2.366079	1.815315	2.787821	1.027598	2.732878	1.224953	1.973279
MW1HK	6.550861	2.689616	1.048007	2.147478	1.34456	1.776809	2.690017	1.505243	1.278511	1.254817
MW1FA	2.745459	2.585569	1.078426	2.353023	1.350216	1.499025	3.246095	1.544332	1.362101	1.374138
MW1CV	10.89823	5.101736	1.41988	3.979459	1.964975	1.910379	10.08978	2.701046	2.085146	1.958867

Table A. 14. MSE result when the magnitude of outlier is 5, $\rho = 0.9$ and $\sigma^2 = 1$.

		$\sigma^2=1$	$\delta^2 = 1$	$k= 5$						
$\rho = 0.9$										
Estimators	15	20	p=3 30	50	100	15	20	p=6 30	50	100
OLS	126.5072	271.4696	34.65088	62.63977	30.79532	123.2034	101.1998	138.9451	87.081	60.85179
MHK	4.352912	3.677233	2.122143	2.807681	2.810617	2.43542	6.208997	1.154124	0.948468	1.181039
MFA	5.692558	6.703001	5.036055	6.257579	6.441657	3.606099	7.991622	3.019502	2.449917	4.900128
MCV	8.814029	35.41234	12.47927	22.00509	16.3293	5.131423	4.076333	14.58039	18.146	24.3679
MWOHK	2.416822	15.10275	0.950408	2.800005	1.557077	1.953958	1.042803	4.97477	2.450251	2.817802
MWOFA	1.386023	3.193777	0.713799	1.759763	1.072253	3.340144	5.884361	3.456652	1.37271	1.644195
MWOCV	2.027627	11.14117	1.918934	3.87503	2.851086	5.072657	1.001649	2.508778	1.316171	2.12965
MW1HK	1.623995	1.824558	1.036167	1.745917	1.356041	1.330356	0.969284	1.372471	1.20335	1.301242
MW1FA	1.699798	2.910655	1.292467	3.029426	1.93844	1.189063	1.234146	1.737194	1.690301	1.9435
MW1CV	4.760389	10.77213	3.479995	14.10629	9.113105	2.111826	6.598669	4.805626	6.191685	8.91862

Table A. 15. MSE result when the magnitude of outlier is 1, $\rho = 0.9$ and $\sigma^2=1$.

		$\sigma^2=1$	$\delta^2 = 2$	$k= 1$						
$\rho = 0.9$										
Estimators	15	20	p=3 30	50	100	15	20	p=6 30	50	100
OLS	60.15271	164.3035	5.995922	28.58405	11.08328	41.28731	54.56183	67.82922	32.08975	19.25399
MHK	15.0158	24.11787	2.994666	4.544599	3.768669	5.285852	11.6366	3.442682	1.829138	1.659665
MFA	5.481039	7.658273	3.775975	5.290664	5.030825	3.555217	10.40464	3.117912	2.21341	3.397448
MCV	13.1017	24.36927	5.564291	12.36736	10.22207	5.304138	13.01331	7.469184	8.429659	10.77422
MWOHK	1.102689	4.565329	1.144569	1.803075	1.253663	1.309924	0.725701	1.66681	1.005094	0.868026
MWOFA	0.850014	1.611645	1.050607	1.214185	1.090659	0.923156	0.77365	1.093237	1.005358	0.973497
MWOCV	1.045919	1.778914	1.039749	1.346937	1.184513	1.158919	0.998108	2.273591	1.001804	0.952369
MW1HK	5.052289	6.922272	1.017332	1.647328	1.062376	5.95326	4.943492	1.726777	1.236639	1.163822
MW1FA	2.966734	3.759169	1.107028	2.49914	1.252363	2.055706	7.558148	1.817698	1.592654	1.521347
MW1CV	14.73634	59.1183	2.276972	12.48869	4.256852	4.619248	0.979807	6.364546	5.419582	4.115383

Table A. 16. MSE result when the magnitude of outlier is 5, $\rho = 0.9$ and $\sigma^2=1$.

	$\sigma^2=1$		$\delta^2 = 2$	$k = 5$						
$\rho = 0.9$			$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	527.637	1521.928	105.7107	483.5593	222.0108	342.6009	373.6179	909.6077	560.8414	374.6429
MHK	13.91261	13.5096	2.248818	2.946788	2.01424	5.385045	21.59889	2.188321	1.107743	1.091263
MFA	7.082656	8.192978	5.699997	7.321729	8.013951	4.3848	12.16324	4.825854	3.566082	7.019679
MCV	10.6589	68.70626	21.56938	73.99654	59.92853	5.608609	5.304321	45.87429	69.907	82.96554
MWOHK	2.258596	79.69117	1.699234	5.503532	1.500308	1.980334	17.53104	16.17064	1.055648	1.015713
MWOFA	1.940267	2.616795	1.07087	1.390117	1.129919	2.453978	1.434539	2.342416	1.047901	0.977348
MWOCV	0.987093	4.205165	1.151851	3.535091	1.650295	3.994004	0.999787	5.030656	1.014609	0.938691
MW1HK	1.68467	2.879232	0.94789	1.221893	0.932133	2.104144	1.009927	2.149335	1.112475	1.094829
MW1FA	2.62133	4.138779	1.450052	3.899909	2.368777	1.624216	5.065427	2.560067	2.344419	2.519887
MW1CV	19.66097	106.0899	5.841289	70.94347	37.28038	3.646932	1.056961	12.66247	23.17113	29.12977

Table A. 17. MSE result when the magnitude of outlier is 1, $\rho = 0.9$ and $\sigma^2 = 25$.

		$\sigma^2 = 25$		$\delta^2 = 1$		$k = 1$					
$\rho = 0.9$				$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	189.6321	203.6889	30.39673	59.78889	49.45181	220.2571	143.4782	145.9341	75.71812	46.45243	
MHK	71.80897	46.91215	8.161308	14.18979	16.63982	24.44261	42.03744	20.04483	9.072692	6.650292	
MFA	6.674479	10.11126	4.566222	6.464691	11.09891	5.807421	17.28241	4.449258	2.722019	4.076516	
MCV	36.1159	33.41734	10.05725	22.67546	23.61071	17.30394	19.96578	18.59855	14.81561	17.42737	
MWOHK	2.848944	94.45488	4.027553	10.70929	1.869212	8.611514	8.576734	13.28203	17.45754	9.173497	
MWOFA	2.343024	7.545964	1.410405	2.891532	4.952978	10.13733	14.68267	7.841918	2.90949	3.530465	
MWOCV	20.38018	96.87238	18.48176	11.52074	16.37591	17.32406	1.038539	35.25014	3.89794	9.37056	
MW1HK	32.3848	32.21795	4.062767	5.082329	11.47392	42.90553	107.6733	12.89313	4.527329	3.096262	
MW1FA	7.344256	13.15276	2.909692	7.947615	13.35765	6.529303	34.55608	6.054003	4.449534	5.779306	
MW1CV	45.12525	138.9506	37.51941	64.81896	258.3124	22.2397	682.6288	101.7068	84.11149	77.10274	

Table A. 18. MSE result when the magnitude of outlier is 5, $\rho = 0.9$ and $\sigma^2 = 25$.

	$\sigma^2 = 25$		$\delta^2 = 1$	$k = 5$						
$\rho = 0.9$			$p=3$			$p=6$				
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	1055.301	3729.398	424.3013	877.12	415.0778	1522.008	1140.489	1675.407	1125.057	795.0179
MHK	62.55967	104.3599	8.441586	15.13494	4.585057	31.14481	75.75627	17.78984	6.973167	3.938416
MFA	7.831965	10.73457	6.656112	8.874587	9.101883	6.621558	15.53936	6.879889	4.487128	8.288072
MCV	10.55268	137.638	47.41144	143.3428	25.46127	7.37892	4.807377	92.01707	112.3607	142.317
MWOHK	46.95445	735.1748	21.64782	23.30419	1.654818	55.82184	1.913275	206.3517	194.5796	81.33386
MWOFA	4.625953	9.345443	4.717865	5.362301	8.630893	31.80715	25.88163	16.90597	7.271707	7.5571
MWOCV	1.545828	427.6899	42.33784	27.98052	17.73619	28.34407	1.001799	25.01311	21.24309	23.54803
MW1HK	11.29145	4.650772	1.452138	1.583129	1.198521	16.59097	3.611912	10.48331	2.177321	1.371074
MW1FA	4.293262	6.534918	3.157593	6.99391	11.63502	3.877036	4.809944	5.90911	5.178143	8.291693
MW1CV	100.4209	127.8516	130.8863	175.2589	588.9448	20.28714	682.5363	139.6685	267.3231	437.4684

Table A. 19. MSE result when the magnitude of outlier is 1, $\rho = 0.9$ and $\sigma^2 = 25$.

	$\sigma^2 = 25$		$\delta^2 = 2$	$k = 1$							
$\rho = 0.9$			$p=3$					$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	2729.977	4903.716	120.7831	606.204	229.0869	934.1785	1331.899	1504.727	714.4881	409.8633	
MHK	1146.169	1100.056	15.4376	40.79741	11.01944	84.84136	174.9516	71.8678	23.03897	9.229579	
MFA	17.13799	23.09101	5.93222	8.052724	7.377228	9.202003	49.95701	11.31331	5.29095	6.859471	
MCV	170.7631	164.8533	21.03143	67.37244	56.15057	23.70355	67.32497	58.96012	67.20421	80.756	
MWOHK	2.563498	106.1756	7.666814	33.52807	5.653172	7.479805	180.0944	35.28072	1.667522	7.382944	
MWOFA	2.23475	3.616066	0.986823	2.04058	1.889062	4.595632	3.336989	5.662092	1.612742	1.233062	
MWOCV	30.78722	49.01888	1.548434	18.74565	9.291132	23.26101	0.999228	17.37332	1.105193	1.597074	
MW1HK	29.88823	49.80876	2.777855	3.357364	1.014992	101.2079	23.87193	14.76361	2.211132	1.173912	
MW1FA	5.44435	16.40974	2.692216	5.067924	4.879605	12.45816	15.93513	7.582643	4.3074	5.626934	
MW1CV	343.1174	171.6384	73.54678	519.7297	306.4952	74.91882	0.984127	243.236	270.1996	252.7105	

Table A. 20. MSE result when the magnitude of outlier is 5, $\rho = 0.9$ and $\sigma^2=25$.

		$\sigma^2 = 25$		$\delta^2 = 2$		$k = 5$					
$\rho = 0.9$				p=3						p=6	
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	14244.9	37023.13	1894.636	9297.181	4412.545	6058.079	6552.54	18371.55	11830.7	7553.105	
MHK	713.7155	225.9144	13.7454	25.9355	5.077425	109.5535	388.8428	50.92033	11.79848	3.147935	
MFA	22.71934	13.77827	7.274328	8.740132	9.939952	8.931722	23.7679	12.05683	6.605278	10.72997	
MCV	303.9412	226.883	129.8638	486.0697	547.5964	17.28436	6.299889	332.5205	522.7024	628.843	
MWOHK	1.254581	2337.277	166.5481	486.8873	92.14039	45.72976	4638.177	839.1423	16.86969	133.4631	
MWOFA	2.039298	18.47529	1.599878	2.94516	5.750588	21.96157	9.534091	24.80309	4.945866	2.002092	
MWOCV	0.966436	84.42966	6.380438	198.3089	63.06507	68.0884	1.000071	22.62072	2.508146	2.99106	
MW1HK	4.966179	360.3977	1.35456	1.149912	0.964005	18.12214	1.880197	45.37042	2.404658	1.028149	
MW1FA	5.328818	8.208455	3.46276	6.404613	8.13143	5.833071	6.499426	8.047788	5.739022	8.785058	
MW1CV	409.2279	1180.449	131.3058	1680.569	1796.096	36.32705	1.037933	215.294	674.8786	1135.991	

Table A. 21. MSE result when the magnitude of outlier is 1, $\rho = 0.9$ and $\sigma^2=100$.

		$\sigma^2 = 100$		$\delta^2 = 1$		$k = 1$					
$\rho = 0.9$				$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	881.9768	541.8924	107.5139	223.7241	199.4628	868.5866	564.1753	567.1471	292.4386	176.9839	
MHK	398.3141	93.80403	22.7121	38.55725	49.26978	91.58265	138.6146	79.61193	34.36435	22.17671	
MFA	10.07632	16.2382	5.5805	6.146349	14.08964	9.036353	32.74721	7.484802	3.963824	5.541082	
MCV	93.95618	38.91694	19.69186	11.82122	69.66183	39.12865	41.07039	50.6114	39.29576	42.52217	
MWOHK	3.724423	108.3476	24.96679	9.131905	1.512301	23.20503	13.70752	51.19726	58.4026	38.29191	
MWOFA	3.161246	9.635775	2.331073	4.43596	6.455099	21.25547	20.9533	13.66176	5.362619	5.597222	
MWOCV	82.33245	76.14787	64.2729	21.25837	33.51711	41.91818	1.039634	132.1539	10.17203	25.98597	
MW1HK	72.57674	97.55294	12.76719	4.011888	5.01259	139.368	413.8702	39.48838	10.73018	4.377299	
MW1FA	11.57537	20.01593	4.16615	4.870265	18.20985	15.87386	57.63007	12.81271	7.357143	8.261371	
MW1CV	476.9154	893.7579	192.0761	106.7035	2381.225	75.78213	3513.427	446.112	503.5301	456.11	

Table A. 22. MSE result when the magnitude of outlier is 5, $\rho = 0.9$ and $\sigma^2=100$.

		$\sigma^2 = 100$		$\delta^2 = 1$	k = 5						
$\rho = 0.9$				p=3					p=6		
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	7145.058	10584.24	1561.812	3416.594	2682.773	5679.139	4034.094	6329.663	4238.588	2955.893	
MHK	667.8549	310.5133	32.12389	52.04906	24.76682	121.9273	276.1437	74.93781	28.13299	13.10447	
MFA	14.49577	12.38235	7.719475	9.829594	15.09401	9.124204	19.61103	10.24084	5.931122	10.00127	
MCV	34.42121	157.252	91.98166	389.9343	2.244014	12.82568	4.849648	225.2045	273.8551	346.7656	
MWOHK	457.1553	1111.972	72.66704	211.9036	1.404199	455.8758	2.189546	1080.298	1029.174	595.1315	
MWOFA	7.862719	10.87359	5.041813	10.13333	9.345842	138.014	32.56018	27.44034	17.39349	15.61707	
MWOCV	2.305022	1017.752	115.383	89.78442	33.13654	61.63351	1.001829	72.43525	116.8601	84.85522	
MW1HK	30.07912	7.295986	1.432508	1.73963	1.420375	100.6208	9.406557	43.10679	3.778235	1.37312	
MW1FA	7.03341	8.424024	3.918282	7.524244	17.12915	7.892059	6.811198	9.280767	6.698386	9.975957	
MW1CV	836.3348	434.4115	161.1312	575.5207	4594.211	60.76885	3520.134	455.6592	1107.145	1779.892	

Table A. 23. MSE result when the magnitude of outlier is 1, $\rho = 0.9$ and $\sigma^2=100$.

		$\sigma^2 = 100$	$\delta^2 = 2$	$k = 1$						
$\rho = 0.9$				$p=3$			$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	10779.03	9274.591	468.1704	2427.271	905.5901	3702.41	5318.201	5941.763	2825.767	1611.052
MHK	4552.576	2331.454	48.57918	160.11	32.98724	333.9683	649.2766	291.9394	92.21062	33.45522
MFA	23.59715	39.56967	6.796556	9.558817	8.447304	15.73907	93.52066	20.40968	8.166864	9.173062
MCV	319.687	166.0625	41.28211	221.3561	150.3472	51.90335	141.5878	140.7188	170.5315	214.0915
MWOHK	6.936278	595.8843	38.95808	147.1555	29.17136	26.81421	1909.478	164.2047	9.886328	66.00614
MWOFA	5.133776	12.1197	1.328667	2.342429	4.775116	13.29873	6.597145	16.32978	3.925348	2.131098
MWOCV	61.84786	10.17768	7.171281	133.3247	56.36616	58.19277	0.999343	32.41168	1.810748	3.24035
MW1HK	27.61934	562.791	4.375146	4.184549	1.046096	261.6095	19.21617	43.57594	3.333151	1.73101
MW1FA	4.945391	73.30373	3.22887	5.231283	5.841509	31.67075	27.87634	13.81284	5.528663	6.914748
MW1CV	893.6859	1823.069	190.6056	2008.985	1370.861	230.8675	1.057955	884.666	1004.049	1083.841

Table A. 24. MSE result when the magnitude of outlier is 5, $\rho = 0.9$ and $\sigma^2=100$.

	$\sigma^2 = 100$		$\delta^2 = 2$	k = 5							
$\rho = 0.9$			p=3					p=6			
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	126857.8	90192.74	7338.8	35158.52	17392.51	23410.74	24983.1	71771.01	46530.32	29489.14	
MHK	6114.138	2083.178	47.45348	95.86089	14.05748	436.8958	1492.831	213.076	48.77392	9.901796	
MFA	14.22753	17.37096	7.654563	9.452727	10.36464	11.99134	28.85736	16.77618	7.911463	11.95831	
MCV	215.0628	21.02272	304.0398	1022.165	1456.02	29.52837	6.00527	700.3725	1125.672	1426.329	
MWOHK	1.680868	6345.835	1163.074	2605.822	704.0513	405.3595	51807.98	4044.215	218.7479	1531.119	
MWOFA	3.973447	66.16426	4.005767	4.278186	20.84528	75.45215	14.42322	63.82756	13.7449	5.269323	
MWOCV	1.646974	187.0355	24.63144	982.4732	338.1198	272.0434	1.000096	78.20272	9.857723	8.976334	
MW1HK	6.660792	2099.369	2.129119	1.161534	0.977664	93.56022	1.94912	255.9512	6.217032	1.075688	
MW1FA	6.006093	9.368537	3.857629	6.691887	8.878318	12.30609	6.552121	10.49547	6.464399	9.827284	
MW1CV	2.400662	1287.138	364.4896	3911.029	5996.833	106.9434	1.030302	554.475	1547.63	3450.195	

Table A. 25. MSE result when the magnitude of outlier is 1, $\rho = 0.95$ and $\sigma^2 = 1$.

	$\sigma^2=1$		$\delta^2 = 1$			$k= 1$				
$\rho = 0.95$			$p=3$			$p=6$				
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	25.82383	41.19524	3.39362	6.228496	2.730412	14.63984	23.64717	19.13465	8.864903	5.409046
MHK	13.25043	16.54608	3.037081	3.890754	2.761658	4.114388	6.065326	2.624338	1.784245	1.150508
MFA	4.654509	5.272737	3.603176	4.711437	3.267899	4.224919	2.578836	2.183303	1.885208	1.63668
MCV	12.6413	13.16602	5.031715	8.193208	5.141858	8.513155	2.518011	5.309036	5.453615	4.57045
MWOHK	1.275797	3.162272	3.194266	1.161559	1.050842	0.874876	2.197604	2.914814	1.308538	0.366792
MWOFA	0.812486	0.99127	2.553326	0.749294	1.028717	2.363752	0.977355	2.721296	1.141235	0.458285
MWOCV	1.389567	1.67409	2.886992	1.332126	1.324415	0.983393	1.074241	3.36888	1.286098	0.732078
MW1HK	4.585303	7.207574	1.184469	2.430481	1.518313	0.846955	4.350564	1.974074	1.563683	1.400147
MW1FA	2.143688	3.237413	1.1763	2.774804	1.583315	1.967395	2.679195	1.783772	1.589058	1.585822
MW1CV	6.260814	7.656113	1.766404	5.362077	2.50308	5.373541	2.122377	3.920138	3.094899	2.702887

Table A. 26. MSE result when the magnitude of outlier is 5, $\rho = 0.95$ and $\sigma^2 = 1$.

$\rho = 0.95$ Estimators	$\sigma^2=1$		$\delta^2 = 1$		$k = 5$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	279.4719	547.6205	60.3767	112.2528	51.77151	349.9056	167.2501	301.0641	158.3323	106.5982
MHK	7.320728	5.924584	2.476098	3.307901	1.855501	2.642498	2.508738	1.708117	1.181514	1.285873
MFA	6.779561	10.58387	6.052716	8.238329	5.689328	4.65081	6.427207	3.911896	3.144159	5.781455
MCV	15.75425	16.78568	17.10231	32.15809	21.95136	26.30061	3.166689	29.11808	31.8497	36.68913
MWOHK	1.989841	91.1761	3.181449	3.223984	1.014883	4.613928	33.60023	8.063168	4.184096	1.781109
MWOFA	1.866834	2.357242	2.176988	0.711882	0.665461	6.024418	1.226935	4.303304	1.573168	1.283149
MWOCV	3.148409	8.944039	3.678817	3.099496	1.266199	1.132469	1.001306	5.66929	1.937606	1.861545
MW1HK	1.384743	3.342104	1.14594	2.069394	1.409693	0.859924	1.504649	1.713588	1.283614	1.407942
MW1FA	1.777839	6.035505	1.546641	4.156659	2.654328	0.928269	2.014345	2.037784	2.062409	2.627218
MW1CV	8.408466	27.81081	4.878808	21.82538	14.00613	3.857063	1.809052	7.959932	9.907966	14.40034

Table A. 27. MSE result when the magnitude of outlier is 1, $\rho = 0.95$ and $\sigma^2=1$.

$\rho = 0.95$ Estimators	$\sigma^2=1$		$\delta^2 = 2$		$k = 1$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	841.4762	353.8744	10.41818	59.47076	18.33196	69.42589	206.1847	170.0379	59.702	36.4549
MHK	525.116	64.20212	3.572263	5.660573	2.578958	9.560363	23.697	6.699209	3.132236	1.650941
MFA	14.65715	9.400612	4.227926	6.62447	4.133967	4.722082	5.640025	3.953429	2.531132	3.290122
MCV	198.9278	33.69516	6.759636	17.36092	10.93875	6.935041	51.69442	14.38617	13.46425	15.3335
MWOHK	2.176763	4.21669	0.643977	1.539465	1.081164	1.031406	1.356512	2.641829	1.034717	1.306398
MWOFA	1.710157	0.736897	0.839529	1.009851	0.994953	0.819202	0.978589	2.045663	0.990052	1.073237
MWOCV	5.550095	0.956141	0.813828	1.062479	1.034669	0.939107	1.006337	3.331246	1.022997	1.075481
MW1HK	337.5195	9.128593	1.124834	2.052447	1.003938	7.193255	24.11493	2.772847	1.485139	1.231356
MW1FA	10.90787	4.974855	1.221684	3.345476	1.541825	2.353828	9.741323	2.186959	1.958109	1.89713
MW1CV	48.08586	34.69837	3.138404	20.4153	6.42913	7.842988	58.50117	8.625096	9.905751	6.722536

Table A. 28. MSE result when the magnitude of outlier is 5, $\rho = 0.95$ and $\sigma^2=1$.

$\rho = 0.95$ Estimators	$\sigma^2=1$		$\delta^2 = 2$		$k = 5$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	2350.055	3396.892	180.2293	847.4789	363.9542	623.2816	1392.116	2251.034	1012.893	688.4754
MHK	60.97959	41.3137	2.957547	3.997476	1.415817	9.387571	16.14445	4.455478	1.665786	1.377372
MFA	17.39606	13.11443	7.49136	11.6094	8.822766	5.0198	9.453529	7.216094	5.431315	10.3094
MCV	5.118492	120.8613	31.9866	131.0137	88.94814	8.920347	2.609493	92.29733	124.5279	141.9749
MWOHK	1.599859	55.2115	1.192205	8.947894	1.072906	6.288286	13.48608	30.27619	3.192942	2.165665
MWOFA	1.752342	1.912767	1.041016	0.938769	0.914378	2.32756	1.908217	4.860294	1.246224	1.134598
MWOCV	8.974721	2.254232	0.904115	2.146373	1.127617	1.75956	1.059667	4.736031	1.524307	1.261527
MW1HK	8.505799	4.221458	1.037036	1.359228	0.887154	2.134472	5.173646	2.991846	1.220691	1.140663
MW1FA	4.608865	6.846335	1.895996	6.241579	3.83419	1.441319	2.985295	3.508205	3.357664	4.169789
MW1CV	17.47705	115.131	9.974033	112.641	61.08927	5.41109	28.11388	18.42848	47.2379	53.75197

Table A. 29. MSE result when the magnitude of outlier is 1, $\rho = 0.95$ and $\sigma^2 = 25$.

		$\sigma^2 = 25$		$\delta^2 = 1$		$k = 1$					
$\rho = 0.95$				$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	999.2906	358.3931	56.37191	142.0389	41.97131	387.6867	368.5126	314.2485	147.7398	85.28695	
MHK	580.6762	64.30083	12.11072	32.6655	6.124974	36.38241	38.3238	41.73131	20.80221	10.01442	
MFA	9.473989	12.82021	5.570507	8.215996	4.760483	7.435305	7.235796	5.37734	3.305849	4.43263	
MCV	179.9932	19.59154	14.27405	32.46996	17.32227	12.23948	30.98663	33.5026	27.19906	26.5877	
MWOHK	16.86255	105.8501	8.828002	5.364027	4.151838	14.17408	26.68373	22.76476	22.10784	11.06987	
MWOFA	2.872284	8.296948	3.526151	0.883192	0.907417	22.04892	5.88243	11.90203	2.819448	3.095967	
MWOCV	42.86754	3.072252	20.20732	10.86047	6.583075	17.48477	1.003982	25.62854	9.409315	9.371926	
MW1HK	156.8758	111.9583	7.490723	9.707602	4.401458	52.29042	71.62082	24.49568	9.37733	4.970811	
MW1FA	13.79738	37.5918	4.461289	12.92268	8.129191	8.710776	9.685646	8.401632	6.35449	8.403688	
MW1CV	319.086	208.9602	49.93453	150.8902	120.4886	27.73462	28.70822	121.8119	149.5669	134.2445	

Table A. 30. MSE result when the magnitude of outlier is 5, $\rho = 0.95$ and $\sigma^2 = 25$.

	$\sigma^2 = 25$		$\delta^2 = 1$	k = 5							
$\rho = 0.95$											
Estimators	15	20	p=3	50	100	15	20	p=6	30	50	100
OLS	4254.415	4186.674	735.8582	1745.721	633.529	1934.47	1853.756	3551.762	2091.73	1401.845	
MHK	388.0266	46.617	16.11772	23.12972	3.291564	32.17283	113.387	37.28434	13.82833	11.73764	
MFA	16.12081	17.47584	9.802296	14.38698	10.16768	14.22548	16.47576	10.12466	7.305834	12.96784	
MCV	151.4128	25.32837	66.52538	338.8395	134.5505	178.1198	5.013751	176.4976	201.2716	239.9618	
MWOHK	226.98	2746.495	61.16221	167.941	52.88547	1.147137	4832.688	257.5632	415.425	163.8115	
MWOFA	11.81899	10.41604	7.216555	1.942878	2.981927	22.14466	8.766033	34.12017	6.145789	9.089997	
MWOCV	1.529359	20.2822	45.88358	55.70636	64.34355	1.014768	0.999096	98.411	45.16865	55.1615	
MW1HK	16.37691	11.62538	3.035574	2.589948	1.412299	2.817082	9.428521	19.43733	2.996385	2.020293	
MW1FA	6.328249	19.43946	5.101708	13.20743	13.86043	4.880699	9.54245	8.780129	7.981711	14.43304	
MW1CV	789.9714	523.8277	151.1165	712.0346	723.8377	30.17038	1.352931	167.6718	467.2578	795.6676	

Table A. 31. MSE result when the magnitude of outlier is 1, $\rho = 0.95$ and $\sigma^2 = 25$.

		$\sigma^2 = 25$		$\delta^2 = 2$		$k = 1$					
$\rho = 0.95$				$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	19070.05	6731.915	220.2151	1329.063	413.8749	1619.763	4894.754	3799.145	1346.352	788.5684	
MHK	12580.87	936.0764	24.44732	70.68407	11.11694	199.4162	330.9396	158.1495	58.92086	20.63066	
MFA	54.00044	39.69095	7.843502	13.02845	8.770113	14.96086	41.25139	16.91322	7.583807	9.866924	
MCV	1230.543	125.7638	30.51048	136.6448	90.01163	34.39432	212.6893	102.1731	118.1572	142.891	
MWOHK	2.13241	311.5215	4.25512	155.2825	3.302584	54.87385	162.3267	186.5969	35.65468	13.87103	
MWOFA	2.516485	6.759218	1.612631	0.904877	1.059003	12.78187	1.980724	9.533691	3.105876	1.839099	
MWOCV	75.6314	1.322414	2.843791	5.172254	3.004916	5.24426	1.63356	11.37808	6.430009	2.579774	
MW1HK	3145.469	226.9333	5.577606	5.13617	1.061284	210.9686	664.0326	28.73393	5.293036	1.525249	
MW1FA	36.25592	70.78598	4.196719	8.014396	7.818588	24.5373	145.3773	11.58059	7.037719	9.351106	
MW1CV	1365.613	303.4819	90.82637	722.9128	561.4252	58.37129	1006.59	281.8364	522.5484	471.1628	

Table A. 32. MSE result when the magnitude of outlier is 5, $\rho = 0.95$ and $\sigma^2=25$.

	$\sigma^2 = 25$		$\delta^2 = 2$	$k = 5$						
$\rho = 0.95$			$p=3$	$p=6$						
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	80790.99	94589.88	3282.983	19670.99	7646.277	8251.99	41604.27	45771.11	21663.91	14113.22
MHK	4993.135	1423.638	27.19961	57.53345	3.877929	146.5518	321.5939	114.0764	28.14336	13.75347
MFA	18.98452	25.44947	11.48187	15.89746	14.01897	23.84336	41.19038	20.65459	12.26829	19.47128
MCV	315.943	732.2949	154.8369	1016.864	1033.531	306.2705	7.62806	566.2045	904.2869	1133.995
MWOHK	3.057405	1932.58	95.78345	2712.438	54.01377	1.011599	4531.042	4494.952	930.6713	295.8712
MWOFA	3.999681	17.54981	4.652975	0.939625	1.606583	3.208269	3.679401	36.09626	9.626825	4.259527
MWOCV	1.705287	0.907627	18.50717	77.28655	18.49971	1.294152	1.011563	8.436842	47.78761	10.76926
MW1HK	33.39422	10.60667	2.101477	1.533669	0.985674	0.660282	81.25126	70.23066	2.910012	1.086436
MW1FA	10.62795	13.26305	5.822414	11.18633	13.32813	14.7345	20.04842	13.42558	9.550822	15.71986
MW1CV	4360.126	1459.199	114.314	2049.875	2963.478	123.8303	3257.364	772.8435	1202.959	2129.452

Table A. 33. MSE result when the magnitude of outlier is 1, $\rho = 0.95$ and $\sigma^2=100$.

	$\sigma^2 = 100$		$\delta^2 = 1$	$k = 1$						
$\rho = 0.95$			$p=3$			$p=6$				
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	1949.825	1392.281	167.5946	425.7147	290.9479	1151.681	1497.892	1219.32	572.632	326.2483
MHK	910.6552	235.7249	35.08063	77.30923	62.28379	59.72123	144.0371	167.4199	81.8865	37.74126
MFA	14.7759	21.36937	9.48963	11.81102	16.58862	13.86572	14.20062	9.672403	5.3274	7.143209
MCV	155.8121	38.66808	25.25449	82.09118	95.91422	24.6248	58.60198	88.31104	73.40916	71.18071
MWOHK	17.47316	172.5222	3.327872	56.78998	1.382402	27.2758	61.27121	75.55056	125.4682	41.83191
MWOFA	4.183038	9.486529	4.138037	1.382604	1.767017	50.16486	9.098551	24.40557	4.492712	6.182162
MWOCV	29.98957	2.962638	11.23104	8.230089	1.097506	2.288937	1.004704	84.11763	41.46235	43.73658
MW1HK	290.1023	368.4812	24.34079	22.24243	4.512635	80.60091	317.8928	73.02817	21.39724	9.369927
MW1FA	19.55394	53.42749	8.97165	15.92025	23.11509	27.42717	24.3532	19.85331	11.65936	13.96042
MW1CV	401.6697	589.7546	121.8554	442.1405	2475.305	284.7736	40.17533	521.5438	747.728	785.1992

Table A. 34. MSE result when the magnitude of outlier is 5, $\rho = 0.95$ and $\sigma^2=100$.

		$\sigma^2 = 100$		$\delta^2 = 1$	$k = 5$						
$\rho = 0.95$				p=3					p=6		
Estimators	15	20	30	50	100	15	20	30	50	100	
OLS	8139.562	18505.19	2128.833	7784.265	2378.37	6280.245	6539.082	13354.63	7906.36	5232.164	
MHK	517.4371	208.7162	135.2271	96.68238	11.27044	103.2581	492.9165	154.8904	56.84874	44.9363	
MFA	15.37106	20.00296	14.57872	17.62987	12.44591	24.89364	23.58708	16.14011	10.50846	17.17069	
MCV	53.89242	233.1464	501.4819	592.6317	385.5987	324.4936	5.699563	415.5191	476.2792	601.5528	
MWOHK	286.1535	10602.4	1.424434	494.3295	425.2781	1.307632	12386.92	1073.081	2854.384	813.6011	
MWOFA	9.829558	19.00209	10.96719	5.65314	6.450237	35.15627	74.45387	58.47754	13.46865	19.29034	
MWOCV	1.126356	173.8669	1.487212	349.3223	344.1636	1.013564	0.999095	301.7036	156.2614	248.0309	
MW1HK	22.57553	15.08387	6.907665	3.400527	1.462077	6.202952	4.109839	69.29589	5.525301	2.203318	
MW1FA	8.712167	13.6411	11.86923	14.58463	16.35497	17.92847	11.6509	15.27553	11.06145	18.08115	
MW1CV	801.2061	605.2319	17.5131	2534.953	3053.984	123.6753	1.061481	527.4641	1742.067	3078.31	

Table A. 35. MSE result when the magnitude of outlier is 1, $\rho = 0.95$ and $\sigma^2=100$.

		$\sigma^2 = 100$		$\delta^2 = 2$		$k = 1$					
$\rho = 0.95$				$p=3$				$p=6$			
Estimators		15	20	30	50	100	15	20	30	50	100
OLS		75328.53	47248.16	902.2884	4906.749	1608.822	6828.32	22468.46	15011.39	5334.685	3106.076
MHK		49990.92	11430.58	92.94098	268.458	39.27536	862.0479	1442.711	640.5664	237.2168	80.63912
MFA		92.45827	61.80823	10.51907	18.53475	11.58417	22.03535	106.1424	33.02574	12.85907	14.86059
MCV		2124.459	367.0019	69.82117	396.7962	252.6456	41.78155	530.6985	216.6261	295.4362	383.1295
MWOHK		2.111507	675.0638	24.92674	1278.791	26.84666	95.1364	1438.098	1607.692	275.2426	141.4281
MWOFA		2.631136	16.82144	3.189571	0.902141	1.689077	8.04898	2.624773	26.92832	7.042517	4.23138
MWOCV		118.9906	16.03513	5.539309	105.3805	14.58608	338.1591	8.893974	16.72344	50.91671	9.448958
MW1HK		12120.6	555.9607	36.08233	8.192975	1.07747	384.1354	5553.963	102.9892	9.974542	1.640762
MW1FA		57.94929	60.53899	5.033165	8.560973	9.353509	58.90033	340.1157	24.13713	9.759565	11.88595
MW1CV		4275.997	1134.765	291.3374	2550.689	2589.523	327.0182	1981.127	882.6768	1741.029	2018.513

Table A. 36. MSE result when the magnitude of outlier is 5, $\rho = 0.95$ and $\sigma^2=100$.

		$\sigma^2 = 100$		$\delta^2 = 2$		$k = 5$					
$\rho = 0.95$				$p=3$				$p=6$			
Estimators		15	20	30	50	100	15	20	30	50	100
OLS		160832.4	497221.7	7144.786	66667.86	30138.33	47159.72	170356.7	178855.7	85406.04	55237.64
MHK		3711.421	9431.299	115.4214	207.8816	12.9329	776.4352	1242.371	467.3165	115.9871	53.27745
MFA		27.01379	36.39156	13.18341	18.12007	15.4004	18.05245	60.92888	30.63776	15.66541	23.04241
MCV		95.17066	1099.631	206.5489	2543.404	2777.764	45.19976	13.95271	1082.522	1875.945	2641.026
MWOHK		110.777	14590.96	57.41252	30067.68	606.8357	2624.004	32245.53	39305.35	7430.664	3393.749
MWOFA		30.05474	32.48628	10.02285	0.969637	3.07721	43.22705	4.186793	92.12556	21.15038	10.55642
MWOCV		6.676694	5.003171	12.46052	743.6549	94.16453	18.31963	1.025291	15.98963	586.0402	47.20802
MW1HK		30.90666	31.9059	2.201375	1.891814	0.992738	72.64792	147.0331	290.5593	9.758318	1.078232
MW1FA		16.42898	16.34993	9.255299	12.05459	14.50885	17.70629	34.1498	18.30636	11.13826	17.37166
MW1CV		1574.343	988.423	5.007853	4987.872	9435.445	257.9842	18703.33	1852.09	2529.85	6040.765

Table A. 37. MSE result when the magnitude of outlier is 1, $\rho = 0.99$ and $\sigma^2 = 1$.

		$\sigma^2=1$		$\delta^2 = 1$		$k= 1$					
$\rho = 0.99$				$p=3$				$p=6$			
Estimators		15	20	30	50	100	15	20	30	50	100
OLS		127.6661	143.53789	15.42542	31.45909	10.9140423	92.27772	72.52779	104.3742	41.82327	24.1967
MHK		43.25446	20.380973	4.519107	5.81017	3.12678367	11.30563	6.787778	6.567588	4.890877	2.855348
MFA		5.154838	5.6096962	3.997579	5.510251	3.64996726	3.879011	3.009305	3.060672	2.247381	2.785992
MCV		12.91354	9.0530619	8.205754	16.61621	10.0801228	7.199637	3.60738	13.4904	15.2052	14.86639
MWOHK		6.534911	21.750098	3.182832	4.002721	0.80484004	13.33589	4.064477	9.864726	2.194803	1.02027
MWOFA		1.798715	6.6234627	3.068539	3.634889	1.52797079	2.2096	1.969037	2.777595	1.52566	0.892079
MWOCV		3.131466	10.738414	3.464713	4.431444	1.5069566	1.632527	3.253497	2.134608	2.035482	1.552852
MW1HK		12.79356	40.339409	2.125158	3.462177	1.99209046	9.802557	10.85456	5.99841	3.429111	2.333591
MW1FA		2.462859	4.1310425	1.47021	3.679838	2.10558631	2.53463	2.252582	2.537668	2.137139	2.21014
MW1CV		6.394196	9.1432537	3.340677	10.25278	5.33549073	2.172205	2.253482	6.953848	7.978221	7.276663

Table A. 38. MSE result when the magnitude of outlier is 5, $\rho = 0.99$ and $\sigma^2 = 1$.

$\rho = 0.99$ Estimators	$\sigma^2=1$		$\delta^2 = 1$		$k=5$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	1659.391	1481.7961	257.9108	486.6457	193.008118	924.6522	474.3652	1631.081	712.0266	460.624
MHK	27.48658	10.060898	4.450641	6.192962	2.3939838	11.42674	12.72547	6.438616	3.036251	2.550156
MFA	10.29644	14.818061	8.615686	14.19244	10.2439098	4.687499	6.215373	6.848063	5.376083	10.88395
MCV	16.08562	13.335358	37.33106	89.51851	65.0753759	10.2644	25.98751	95.55507	115.455	128.5603
MWOHK	34.6577	44.5411	5.76734	8.813913	0.91534027	23.27573	22.64896	98.19795	8.763558	3.323479
MWOFA	4.046228	14.607663	2.771762	3.087314	1.66930736	5.613601	1.894207	5.292265	3.516427	2.431987
MWOCV	6.060413	35.81867	5.330212	9.201382	3.88342508	1.349749	1.024719	2.587914	8.18818	7.652781
MW1HK	2.507044	4.6174046	1.754706	3.202922	1.8326921	4.14049	11.42116	3.730993	2.08595	1.968542
MW1FA	2.108547	7.2380449	2.236496	6.700797	4.73735131	1.564868	2.860879	2.861728	2.903728	4.169398
MW1CV	6.394848	52.296653	9.752965	40.51817	36.656704	2.240009	7.844413	13.57472	36.13333	51.13174

Table A. 39. MSE result when the magnitude of outlier is 1, $\rho = 0.99$ and $\sigma^2=1$.

$\rho = 0.99$ Estimators	$\sigma^2=1$		$\delta^2 = 2$		$k=1$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	808.4389	2174.3502	48.84546	303.6173	76.4078836	295.7185	784.7439	1060.703	266.3795	168.2606
MHK	215.9892	502.95257	6.888728	11.64747	3.57874084	30.43715	14.33396	20.62239	12.60557	4.235901
MFA	8.811931	14.44023	5.010209	10.32725	5.89241134	5.009915	6.321715	6.33191	3.277857	5.425092
MCV	13.78865	20.612256	12.48779	36.90198	29.3926377	10.46468	20.32037	36.48958	43.509	53.1768
MWOHK	2.918086	28.789136	1.542335	1.073562	0.76791654	7.664978	3.770215	20.19266	2.059376	1.429724
MWOFA	1.244609	2.5420954	1.298793	0.793216	0.91419104	3.069071	1.276056	2.433584	1.129262	1.0824
MWOCV	1.829907	5.066804	1.242597	0.92044	0.85151717	1.609371	0.999049	1.452911	1.126845	1.010496
MW1HK	88.69094	32.720162	2.830204	3.767052	1.20720493	33.99575	122.0694	8.203923	3.667731	1.7711
MW1FA	8.471116	9.8860848	1.655745	5.643643	2.51037192	3.551531	9.460104	3.108573	2.752116	2.743177
MW1CV	34.04981	30.575216	6.430143	43.89764	18.8900052	3.621044	16.75504	16.1962	32.31122	25.15803

Table A. 40. MSE result when the magnitude of outlier is 5, $\rho = 0.99$ and $\sigma^2=1$.

$\rho = 0.99$ Estimators	$\sigma^2=1$		$\delta^2 = 2$		$k=5$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	12904.86	33630.163	767.5325	4813.744	1391.39814	2351.233	4333.814	13751.77	4329.779	3081.567
MHK	103.8156	3650.1448	7.054328	12.73819	2.2849499	36.85315	31.70901	24.14376	7.753889	3.269431
MFA	28.67965	37.861472	12.90643	28.63851	26.0240214	6.558158	12.71349	18.10106	13.25705	27.73553
MCV	29.62512	75.602932	69.57953	331.8784	284.634782	13.54961	38.60281	254.486	353.8021	548.4948
MWOHK	15.25958	616.45238	3.630683	13.2149	1.78727994	55.94042	26.51373	433.7134	32.38608	12.79438
MWOFA	3.005539	6.7768831	1.322997	0.876852	0.98812224	6.01597	4.042154	6.0627	2.076724	1.699638
MWOCV	15.26281	5.9242634	1.638349	2.257668	1.00493884	2.765488	1.097043	2.835432	2.470978	1.223807
MW1HK	24.55025	4.8128919	2.152631	3.159681	0.99324885	15.83778	25.77349	15.38367	2.307955	1.553859
MW1FA	17.66225	18.063083	3.732081	15.95006	11.3107064	2.216838	4.483323	6.333785	6.711075	10.05828
MW1CV	41.0529	10.699765	23.22847	178.4437	178.624793	3.597277	26.17193	36.601	154.2848	228.087

Table A. 41. MSE result when the magnitude of outlier is 1, $\rho = 0.99$ and $\sigma^2 = 25$.

$\rho = 0.99$ Estimators	$\sigma^2 = 25$		$\delta^2 = 1$		$k = 1$		$\sigma^2 = 25$		$\delta^2 = 1$		$k = 1$	
	15	20	p=3 30	50	100	15	20	p=6 30	50	100	15	20
OLS	15565.68	1847.1344	272.7699	502.1547	178.064546	1915.269	1894.615	1717.806	722.5572	390.2003	1915.269	1894.615
MHK	12054.12	202.06151	38.46128	48.54864	16.9591595	217.2732	84.44844	128.6277	91.02192	36.73135	217.2732	84.44844
MFA	31.58627	13.498907	8.007942	12.01258	8.10697728	9.063837	9.670832	8.025939	4.960443	7.770178	9.063837	9.670832
MCV	307.1938	20.927537	31.56131	19.65741	45.8754284	29.81412	58.94192	75.03203	101.2721	94.49407	29.81412	58.94192
MWOHK	320.9031	131.66134	32.35545	26.28627	2.69652047	111.0206	43.18102	108.309	54.93954	26.66172	111.0206	43.18102
MWOFA	8.051523	52.674882	3.832097	4.447041	4.8238897	34.82662	13.76133	20.22192	12.58425	8.462131	34.82662	13.76133
MWOCV	218.1349	144.88408	25.90767	38.9651	31.1628784	10.03633	5.16331	11.42696	42.45041	47.68153	10.03633	5.16331
MW1HK	5811.128	117.24243	26.59296	20.35364	14.9465662	309.0977	348.4551	102.612	38.39206	18.75104	309.0977	348.4551
MW1FA	82.04523	63.585691	10.67593	25.18028	22.3313449	12.15343	12.74473	14.67112	12.59647	18.33295	12.15343	12.74473
MW1CV	280.2365	136.20189	68.87858	57.66919	225.737231	14.82127	13.01903	115.1357	294.7582	423.6468	14.82127	13.01903

Table A. 42. MSE result when the magnitude of outlier is 5, $\rho = 0.99$ and $\sigma^2 = 25$.

$\rho = 0.99$ Estimators	$\sigma^2 = 25$		$\delta^2 = 1$		$k = 5$		$\sigma^2 = 25$		$\delta^2 = 1$		$k = 5$	
	15	20	p=3 30	50	100	15	20	p=6 30	50	100	15	20
OLS	59255.5	8686.2815	3439.462	6806.986	2903.68441	12839.46	9522.68	19002.54	9636.743	6097.519	12839.46	9522.68
MHK	6837.578	59.016973	47.33147	62.17146	19.2075016	286.0053	328.9041	159.9744	72.06904	36.84313	286.0053	328.9041
MFA	30.37534	29.136739	22.20965	37.10117	35.3203584	13.8247	21.43576	25.21819	20.91017	39.0855	13.8247	21.43576
MCV	2.111806	19.903198	142.8937	599.9597	312.973976	26.43286	258.8128	422.2282	623.1987	870.4008	26.43286	258.8128
MWOHK	83.10776	511.80249	323.4586	444.6857	82.6326219	881.7842	206.7246	1604.101	659.1043	256.8421	881.7842	206.7246
MWOFA	38.35216	87.403496	7.167917	17.04216	21.2248136	79.06638	32.6909	49.43235	38.26429	35.97213	79.06638	32.6909
MWOCV	1.997444	677.99441	63.46812	269.1523	102.035796	5.090194	64.64685	17.92263	137.4902	223.6985	5.090194	64.64685
MW1HK	166.3162	38.314	16.0099	15.73694	10.4121349	153.8378	137.273	63.51057	17.44461	13.49115	153.8378	137.273
MW1FA	24.45946	57.535224	16.66354	40.30498	50.5541756	6.667199	11.29294	22.20024	23.56812	50.66095	6.667199	11.29294
MW1CV	9.941057	729.64391	149.3168	878.6921	719.568943	20.95274	59.98041	152.2408	985.8075	1733.617	20.95274	59.98041

Table A. 43. MSE result when the magnitude of outlier is 1, $\rho = 0.99$ and $\sigma^2 = 25$.

$\rho = 0.99$ Estimators	$\sigma^2 = 25$		$\delta^2 = 2$		$k = 1$		$\sigma^2 = 25$		$\delta^2 = 2$		$k = 1$	
	15	20	p=3 30	50	100	15	20	p=6 30	50	100	15	20
OLS	53579.8	72353.994	1036.698	6375.459	1633.88531	7045.549	16009.1	23911.13	6087.991	3697.42	7045.549	16009.1
MHK	11999.36	18751.983	89.47787	214.1301	33.7722895	705.1174	273.1463	509.8523	295.7458	75.76005	705.1174	273.1463
MFA	116.7467	97.692433	15.4521	37.93083	25.049107	17.60663	44.7916	36.96434	15.98643	25.30042	17.60663	44.7916
MCV	177.4378	147.2392	64.04208	210.7073	215.920772	42.64698	94.45535	154.852	308.3182	465.4591	42.64698	94.45535
MWOHK	1.385274	5764.8431	35.917	166.437	16.4853023	28.73833	24.28609	786.7229	439.3615	130.2927	28.73833	24.28609
MWOFA	2.638944	9.4459336	1.652815	1.067202	1.21156713	12.16174	5.382958	11.42322	3.684085	4.552238	12.16174	5.382958
MWOCV	159.1514	16.412987	7.007048	31.12986	3.91081192	8.518779	1.157686	7.123423	9.070204	2.927951	8.518779	1.157686
MW1HK	3272.942	283.32792	26.16123	44.27533	2.47648227	925.775	5295.273	152.5636	31.34505	7.302264	925.775	5295.273
MW1FA	632.4083	94.876497	14.78775	26.69592	28.3277478	31.00867	157.92	31.42417	22.14809	32.08761	31.00867	157.92
MW1CV	2556.195	221.00518	141.4856	749.1127	1057.26006	36.16642	93.21713	252.1894	887.8208	1285.297	36.16642	93.21713

Table A. 44. MSE result when the magnitude of outlier is 5, $\rho = 0.99$ and $\sigma^2=25$.

$\rho = 0.99$ Estimators	$\sigma^2 = 25$		$\delta^2 = 2$		$k = 5$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	489495.7	319439.31	13969.51	92780.13	28688.2594	48714.09	135851.2	282234.2	93878.64	64318.62
MHK	42660.94	3955.1803	106.853	242.7671	22.4489223	903.1715	1120.305	629.6828	203.6407	60.39918
MFA	59.3304	67.637213	32.8901	61.20093	67.2174899	25.27371	67.54329	65.71578	44.3061	79.29415
MCV	264.1925	290.05736	313.2069	1565.789	2610.0451	48.96998	271.8862	1087.041	1841.778	3623.374
MWOHK	19.94226	157753.01	560.6876	4121.31	295.757151	284.6341	999.046	16262.93	10959.51	3269.257
MWOFA	21.13034	58.86795	2.453543	2.050708	1.95695743	63.63284	30.91069	33.38679	11.8054	16.06499
MWOCV	2.068396	90.901086	18.43237	146.6839	33.372276	14.64959	1.331261	24.01114	28.69464	11.04487
MW1HK	1147.882	351.84168	14.32309	34.91813	1.18036512	496.664	592.8397	465.7087	13.85906	2.749908
MW1FA	50.57566	91.521039	22.99212	44.35047	55.9670143	17.00932	37.65657	47.15915	37.03663	67.40713
MW1CV	110.7917	230.17064	227.0603	2603.985	5026.89538	40.55808	81.10093	368.9892	2202.304	4290.076

Table A. 45. MSE result when the magnitude of outlier is 1, $\rho = 0.99$ and $\sigma^2=100$.

$\rho = 0.99$ Estimators	$\sigma^2 = 100$		$\delta^2 = 1$		$k = 1$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	60505	7198.9921	1044.528	1894.802	771.095055	7598.63	7702.419	6658.426	2810.557	1501.548
MHK	47271.33	995.77637	137.258	177.1135	55.400485	858.3231	325.0156	513.5613	361.2602	143.5622
MFA	63.56002	33.608666	14.39011	20.61622	14.059222	17.20824	22.36556	17.06862	10.15972	15.50334
MCV	515.8509	90.323663	76.02245	11.32032	134.034702	55.50242	114.8231	162.2027	247.2647	261.3928
MWOHK	420.4219	941.4472	185.6584	63.44058	22.5674268	383.5103	161.9474	347.1893	175.7217	122.3305
MWOFA	14.92791	100.68622	5.675011	6.270499	14.0437025	92.90264	31.08018	49.50175	27.01457	25.9566
MWOCV	432.3686	1118.742	94.52774	58.04458	83.6090351	29.08919	6.873278	35.61551	148.5789	193.5755
MW1HK	16979.77	1586.5734	75.31132	44.67482	31.2578396	1029.963	1043.564	318.8553	109.5812	56.30274
MW1FA	271.4164	177.20901	23.0624	37.46756	40.7719325	38.23026	57.93217	49.09069	33.34842	46.08535
MW1CV	541.1443	251.25852	256.8965	6.682781	602.488627	46.02066	10.08839	341.5912	1017.871	1730.426

Table A. 46. MSE result when the magnitude of outlier is 5, $\rho = 0.99$ and $\sigma^2=100$.

$\rho = 0.99$ Estimators	$\sigma^2 = 100$		$\delta^2 = 1$		$k = 5$					
	15	20	$p=3$		50	100	15	20	$p=6$	
			30						30	100
OLS	220007.7	73029.734	12499.36	37068.47	9086.28934	49582.99	40904.99	71092.37	36542.03	22905.19
MHK	29854.17	385.97781	183.6435	311.511	66.0811116	1147.393	1400.276	651.3677	298.6074	144.5624
MFA	39.88598	53.317821	32.47233	57.76034	53.37035	25.73044	41.00567	46.04669	35.32739	62.57887
MCV	2.132015	96.189017	275.8827	2310.768	1286.15612	63.37882	463.3466	823.5264	1282.257	2017.81
MWOHK	95.7696	1160.9314	2447.77	2191.791	106.722432	6186.344	463.6381	5863.839	2734.47	1785.834
MWOFA	50.06649	137.06703	12.73432	23.02696	60.702448	172.0437	140.4452	119.8204	68.36277	113.0421
MWOCV	2.00529	1905.9793	225.1547	125.715	888.812359	11.98874	363.1361	40.16725	407.1694	655.071
MW1HK	628.9954	179.73553	26.57267	41.22816	9.70278123	565.9777	988.9381	232.531	47.09856	24.96094
MW1FA	32.88885	80.762205	24.26323	55.88705	68.8495595	19.15731	23.30001	50.87764	40.02262	78.53103
MW1CV	3.373225	2717.0175	554.7267	2515.303	3877.26809	48.48892	8.812618	391.673	2597.74	5231.894

Table A. 47. MSE result when the magnitude of outlier is 1, $\rho = 0.99$ and $\sigma^2=100$.

	$\sigma^2 = 100$		$\delta^2 = 2$		$k = 1$					
$\rho = 0.99$			$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	172336.6	218741.53	4242.087	25150.33	5011.94582	28149.79	84947.62	94550.78	24165.36	14593.74
MHK	99291.99	18370.992	375.4926	841.5959	129.206595	2819.622	1179.991	2051.416	1190.044	302.3055
MFA	102.8043	182.39411	25.49352	62.96733	38.4510542	37.03218	131.7797	81.07015	31.63878	45.4093
MCV	238.5031	274.29944	135.4878	454.369	540.945019	77.93387	312.6322	307.1308	645.0462	1090.644
MWOHK	1.46408	37846.993	254.2985	2192.408	151.800333	104.173	114.5338	2245.434	4588.531	1132.133
MWOFA	2.936593	29.07973	1.873563	1.735818	1.87449524	44.63336	14.02002	21.35695	8.632243	12.28559
MWOCV	312.7457	33.141989	17.59861	133.3229	10.5936389	15.6983	1.022455	31.98828	27.14456	10.12424
MW1HK	19642.71	3290.2805	73.41311	83.49957	3.51601035	3735.046	12326.58	473.3527	96.65276	14.15839
MW1FA	113.016	415.55342	22.73921	32.34041	39.5590311	108.0012	334.2452	80.91154	38.89081	50.31635
MW1CV	2035.941	1160.3885	418.9399	2564.572	2689.46653	152.8725	277.2733	867.1924	2212.489	3882.588

Table A. 48. MSE result when the magnitude of outlier is 5, $\rho = 0.99$ and $\sigma^2=100$.

	$\sigma^2 = 100$		$\delta^2 = 2$		$k = 5$					
$\rho = 0.99$			$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	989187.2	1828907.6	59289.9	362444.6	117667.956	193791	560380.2	1103511	370911.1	252382.2
MHK	10705.07	12259.43	397.6916	951.511	96.7407202	3605.841	4753.343	2546.315	833.2792	242.7852
MFA	114.6812	132.28436	45.4304	74.52474	84.6410432	45.34551	117.0019	106.6811	63.95705	105.8774
MCV	122.0958	852.47777	453.1336	3340.359	5812.48078	80.84566	408.9078	1927.973	3385.653	7291.641
MWOHK	1596.308	425359.02	7615.163	54136.74	4873.17101	2093.126	6483.826	45591.07	117011.3	28329.46
MWOFA	19.73669	118.22813	4.218159	8.653425	5.16387681	294.0902	50.14626	118.5831	38.4848	37.15015
MWOCV	740.6258	187.16006	35.86909	689.4144	247.856049	32.05722	1.181407	71.69216	71.20407	26.47444
MW1HK	1369.27	707.86331	9.710728	292.952	1.18180274	1632.199	1506.138	2016.488	42.56008	3.331639
MW1FA	568.4748	135.09406	30.94463	48.68673	63.5119069	45.98926	74.8993	77.30638	47.98183	80.30442
MW1CV	2466.444	926.77152	530.9558	7191.053	20064.3749	143.2215	377.0958	996.4182	5498.079	13956.54

Table A. 49. MSE result when the magnitude of outlier is 1, $\rho = 0.999$ and $\sigma^2 = 1$.

	$\sigma^2=1$		$\delta^2 = 1$		$k = 1$					
$\rho = 0.999$			$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	8303.227605	1279.85914	157.214673	308.414773	100.615548	871.581593	662.6011	1089.68668	410.3011419	233.6186
MHK	5761.622479	72.93241011	16.8545138	22.8453092	6.88489588	97.5854196	57.927072	54.1093914	39.55428447	13.83003
MFA	7.249734783	5.326431811	4.22031184	5.97484275	3.88494454	5.00614585	4.3828406	3.73117346	2.472490917	3.14258
MCV	14.83438539	5.00818396	8.29823286	18.6783798	19.6797994	5.05369893	5.4730862	12.7019202	20.65279973	35.93708
MWOHK	1131.198898	47.32438608	6.9367054	11.210078	2.05920653	105.447874	82.59372	49.5489606	9.597889566	8.831464
MWOFA	6.84185895	7.091538572	2.56851222	2.54290901	1.80371918	7.1886976	3.6122304	3.83600981	1.583029876	2.948874
MWOCV	1.620738408	7.764685458	3.10951628	4.25836568	2.03175356	3.36048793	2.5398131	2.56065552	6.047616682	2.513402
MW1HK	763.807149	49.822108	11.5786054	12.492791	4.55367025	83.6426718	104.66811	52.4331936	24.54607326	11.15195
MW1FA	5.680134368	4.913648455	1.86923785	4.04396615	2.50920678	4.79672128	4.5907608	3.90839758	2.837662263	2.780416
MW1CV	5.481267462	6.942400438	4.46459017	12.4878878	13.6727719	1.74896833	1.8780827	5.10214172	13.43293291	22.55933

Table A. 50. MSE result when the magnitude of outlier is 5, $\rho = 0.999$ and $\sigma^2 = 1$.

	$\sigma^2=1$		$\delta^2 = 1$		$k = 5$					
$\rho = 0.999$			$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	45837.98392	25006.73251	2019.94671	8526.90287	1825.74835	8671.72907	5322.1209	16914.1977	6836.91717	4384.652
MHK	1390.819515	18.48541743	10.5434849	20.9700548	6.8935365	94.8868412	86.39473	38.9263796	15.68919674	9.369043
MFA	18.06760236	16.35360553	11.4737383	27.0096106	15.704472	5.38746093	11.371199	10.9955626	8.41979052	15.76482
MCV	2.863550625	30.6205395	23.9309047	211.540641	178.545705	15.0961633	3.4013571	118.64484	182.34649	365.2643
MWOHK	210.1264776	84.91391171	71.8636085	281.871375	6.57847935	122.962776	1050.2328	639.081263	25.69829452	143.1046
MWOFA	10.83571714	22.47075742	3.20511617	5.09304179	2.50486231	13.4004619	40.204361	7.57879387	4.131356481	7.55261
MWOCV	1.981420981	24.8425059	6.16445424	4.40464535	4.28338957	2.5660034	1.6479117	4.5286754	17.3286693	5.268753
MW1HK	71.97976572	13.04749043	6.08666658	11.1452218	3.87849891	24.1266266	37.484335	30.2072126	8.694087861	5.478849
MW1FA	4.860476438	10.46854345	2.6873199	13.3702312	6.49494496	2.07402654	1.7759816	3.72633682	3.539782164	5.299925
MW1CV	8.388879343	10.89786568	7.62899118	34.7895498	47.3952658	1.72569872	5.5835339	10.3552	44.58079829	92.7902

Table A. 51. MSE result when the magnitude of outlier is 1, $\rho = 0.999$ and $\sigma^2 = 1$.

		$\sigma^2=1$	$\delta^2=2$	$k=1$						
$\rho=0.999$			$p=3$				$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	53873.2	27052.6	513.9	2949.7	718.6	2705.8	5917.3	11622.0	2497.3	1624.0
MHK	39475.8	7733.4	38.2	68.8	10.9	277.0	93.1	178.7	122.2	23.0
MFA	17.6	21.0	5.6	13.3	7.4	6.1	12.2	9.2	4.0	6.7
MCV	33.9	12.8	11.9	27.6	59.0	7.6	14.4	18.5	70.7	99.0
MWOHK	39.6	1604.4	1.7	3.3	1.0	36.6	240.4	218.1	11.7	1.2
MWOFA	3.8	5.6	1.3	1.3	0.9	7.5	5.9	3.2	1.8	1.4
MWOCV	2.0	4.0	1.2	1.4	0.9	3.1	5.6	2.2	2.5	1.0
MW1HK	31575.8	408.9	21.4	12.7	2.9	321.1	109.4	70.7	25.3	5.6
MW1FA	44.7	23.8	2.2	8.4	3.5	5.7	8.4	4.3	3.4	3.4
MW1CV	27.9	14.5	6.7	34.6	43.1	3.3	37.2	9.2	35.4	66.5

Table A. 52. MSE result when the magnitude of outlier is 5, $\rho = 0.999$ and $\sigma^2 = 1$.

		$\sigma^2=1$	$\delta^2 = 2$	$k = 5$						
$\rho = 0.999$			$p=3$					$p=6$		
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	99006.9	175988.0	7025.1	48178.7	12842.4	22035.6	43182.3	148769.5	39694.5	29519.2
MHK	1733.1	480.2	23.6	62.3	13.4	339.3	269.0	166.7	53.4	22.3
MFA	31.7	39.5	21.2	82.3	66.6	8.4	20.8	50.8	32.3	63.4
MCV	11.1	76.2	107.3	353.4	665.9	21.1	161.0	292.6	513.8	1098.1
MWOHK	219.3	9974.6	5.3	25.0	2.4	43.0	1269.8	4553.6	182.0	2.9
MWOFA	8.4	11.7	1.4	2.1	1.1	15.9	12.6	13.4	3.5	3.8
MWOCV	3.0	25.5	1.5	3.3	1.8	4.6	1.9	3.1	5.9	1.2
MW1HK	61.0	64.3	11.3	18.3	3.6	167.9	367.3	99.9	10.5	5.7
MW1FA	19.4	43.0	7.2	50.5	29.5	2.7	14.2	12.8	12.9	19.8
MW1CV	12.3	22.0	15.9	67.4	161.4	4.8	4.3	21.5	119.0	260.3

Table A. 53. MSE result when the magnitude of outlier is 1, $\rho = 0.999$ and $\sigma^2 = 25$.

		$\sigma^2 = 25$	$\delta^2 = 1$	$k = 1$							
$\rho = 0.999$			$p=3$						$p=6$		
Estimators	15	20	30	50	100	15	20		30	50	100
OLS	160747.7	20014.7	2329.0	6883.7	1941.5	18271.0	10645.0		18008.1	7189.4	3802.5
MHK	127556.4	1580.5	152.5	955.5	166.1	2185.5	810.8		1203.7	924.0	326.6
MFA	35.6	21.0	12.7	20.2	13.6	11.3	30.7		11.2	7.1	10.5
MCV	8.8	24.1	39.8	30.5	38.2	12.6	29.9		70.5	135.4	220.1
MWOHK	14333.0	447.2	827.0	2618.0	8.1	713.8	421.9		1734.8	231.0	896.9
MWOFA	24.1	215.5	8.5	20.4	8.3	53.4	175.2		45.4	33.6	28.4
MWOCV	2.0	45.0	30.5	40.4	7.3	16.2	582.3		17.2	57.7	20.4
MW1HK	64076.6	394.2	708.0	279.1	209.5	2835.6	4542.4		1106.3	313.1	113.5
MW1FA	119.6	164.0	28.8	103.9	73.4	19.0	68.8		26.0	24.9	36.8
MW1CV	145.6	56.2	26.4	140.5	176.6	4.8	225.8		25.9	148.6	274.7

Table A. 54. MSE result when the magnitude of outlier is 5, $\rho = 0.999$ and $\sigma^2 = 25$.

$\sigma^2 = 25$		$\delta^2 = 1$		$k = 5$						
$\rho = 0.999$				$p=3$			$p=6$			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	231738.2	89228.4	19081.4	54264.7	25887.1	124194.4	92009.8	196639.2	93433.5	58283.3
MHK	18039.1	302.5	163.6	245.0	141.7	2698.7	5006.4	1127.2	523.2	190.0
MFA	61.4	45.7	52.8	96.4	119.1	24.2	41.4	68.9	63.4	105.2
MCV	74.6	20.6	96.9	533.5	670.6	38.8	459.5	477.9	747.5	1731.8
MWOHK	585.2	2014.1	5014.7	12035.8	3.4	8064.7	16115.4	34014.6	1962.3	19747.5
MWOFA	112.5	505.9	8.2	71.3	13.8	158.0	138.0	157.4	178.8	66.6
MWOCV	2.3	259.6	119.8	110.0	1.9	11.4	110.4	23.3	182.4	86.9
MW1HK	710.0	380.9	73.8	303.6	229.8	874.5	1069.4	1176.9	121.3	159.3
MW1FA	69.0	204.6	82.6	205.7	328.4	12.0	23.3	69.3	93.8	216.3
MW1CV	15.0	578.4	22.9	268.5	978.8	6.2	6.6	60.6	280.0	642.9

Table A. 55. MSE result when the magnitude of outlier is 1, $\rho = 0.999$ and $\sigma^2 = 25$.

$\delta^2 = 2$	$\sigma^2 = 25$		$k = 1$							
$\rho = 0.999$			p=3					p=6		
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	466643.7	440102.4	11716.5	62008.3	19448.8	63766.7	152193.3	263012.4	57346.6	35883.2
MHK	323486.3	78349.5	1058.7	1411.0	383.6	6533.8	1950.1	4532.5	3036.5	571.9
MFA	101.7	211.5	28.6	104.9	132.1	26.3	146.8	86.8	36.9	57.8
MCV	113.0	128.6	52.9	151.4	394.0	20.3	178.3	84.3	243.1	649.0
MWOHK	464.4	13583.2	49.0	813.1	1.9	675.2	3920.8	19873.9	1872.9	42.0
MWOFA	8.3	94.0	1.8	3.6	1.7	37.2	44.3	43.3	11.9	14.2
MWOCV	6.1	54.9	4.6	20.7	15.5	11.2	38.9	6.1	16.3	5.5
MW1HK	111689.8	3457.4	255.0	368.9	23.9	7675.0	6131.3	925.7	228.7	86.5
MW1FA	852.9	585.8	65.1	180.2	314.9	56.1	90.6	102.1	91.5	134.2
MW1CV	1.1	273.2	109.4	302.8	1706.2	17.5	56.2	103.2	275.3	629.5

Table A. 56. MSE result when the magnitude of outlier is 5, $\rho = 0.999$ and $\sigma^2 = 25$.

$\delta^2 = 2$	$\sigma^2 = 25$			$k = 5$						
$\rho = 0.999$				p=3				p=6		
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	1799463.5	5340750.9	143687.6	832850.5	330589.4	462546.7	1374729.6	3063898.6	863866.6	620612.4
MHK	67282.2	15051.1	648.6	1070.3	836.2	8037.9	10830.8	4207.1	1489.2	517.7
MFA	195.1	392.3	122.4	301.7	578.7	58.1	190.2	276.4	203.6	366.8
MCV	230.7	321.5	246.8	3140.7	5.9	42.2	153.5	894.4	1462.5	3947.9
MWOHK	3798.7	339898.8	1267.5	24630.3	1.8	2483.9	93798.1	419961.7	44179.5	939.2
MWOFA	41.9	393.6	2.5	8.5	2.9	221.3	204.6	151.8	82.4	55.8
MWOCV	7.0	221.0	97.0	97.3	93.7	13.9	1.1	6.2	60.9	14.3
MW1HK	1417.0	610.8	302.4	295.7	2.4	2145.4	647.4	4129.4	212.7	86.6
MW1FA	207.2	597.9	145.1	353.2	757.3	37.5	168.3	244.3	236.4	474.9
MW1CV	147.6	135.3	363.1	2110.1	14677.5	23.5	246.4	280.3	1190.4	2855.8

Table A. 57. MSE result when the magnitude of outlier is 1, $\rho = 0.999$ and $\sigma^2 = 100$.

$\delta^2 = 1$	$\sigma^2 = 100$			$k = 1$						
$\rho = 0.999$			p=3					p=6		
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	212074.5	126990.8	10971.9	19013.5	7533.3	72612.9	42720.5	69796.2	27996.9	14663.9
MHK	129269.0	5801.6	1452.3	1394.7	371.3	8665.5	3130.8	4791.3	3669.9	1302.9
MFA	38.5	81.7	24.4	42.8	29.2	25.5	91.6	30.3	19.8	27.8
MCV	11.2	21.2	45.7	11.6	262.9	22.8	87.7	129.8	297.6	517.8
MWOHK	2518.9	7333.0	1145.8	632.6	10.5	2192.3	2427.7	4136.7	661.9	5063.3
MWOFA	67.1	882.6	7.0	36.2	13.5	249.1	801.8	190.1	138.0	70.1
MWOCV	3.1	13.9	117.0	229.4	2.2	57.7	2414.8	47.5	196.5	52.4
MW1HK	31292.8	1063.7	529.7	450.9	284.5	9864.3	8310.0	2828.8	684.6	375.0
MW1FA	260.8	404.2	101.8	205.7	203.3	72.5	498.9	141.3	116.2	179.9
MW1CV	207.1	275.4	169.1	8.8	638.0	18.4	1206.3	101.5	265.7	581.2

Table A. 58. MSE result when the magnitude of outlier is 5, $\rho = 0.999$ and $\sigma^2 = 100$.

$\delta^2 = 1$	$\sigma^2 = 100$		k = 5							
$\rho = 0.999$			p=3				p=6			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	2336325.2	749526.4	78800.5	316086.9	143406.8	482862.8	274187.5	707916.8	354511.8	219454.1
MHK	337285.2	1624.3	1872.3	955.0	811.7	10857.2	9650.4	4612.0	2164.8	758.3
MFA	155.6	123.7	116.8	239.8	402.6	59.3	132.3	159.6	142.7	234.4
MCV	2.1	185.6	97.2	120.2	3.5	47.7	11.5	748.0	1443.7	2956.7
MWOHK	8558.7	9422.5	73725.5	6788.5	1.4	42093.9	93788.9	75043.7	8121.3	122669.8
MWOFA	338.1	988.3	27.4	254.0	116.8	572.5	577.9	546.9	440.1	124.7
MWOCV	2.0	418.3	5.5	162.6	1.7	45.6	33.9	94.9	971.1	146.0
MW1HK	12500.0	2101.2	402.4	612.9	45.5	2699.3	620.2	4842.8	543.0	606.8
MW1FA	220.9	444.0	232.6	328.6	1005.0	49.5	47.5	239.0	242.9	544.2
MW1CV	54.0	2195.6	62.0	1523.8	13009.6	20.5	1.0	194.5	918.8	2676.4

Table A. 59. MSE result when the magnitude of outlier is 1, $\rho = 0.999$ and $\sigma^2 = 100$.

$\delta^2 = 2$	$\sigma^2 = 100$		k = 1							
$\rho = 0.999$			p=3				p=6			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	4808707.9	1464284.1	44330.2	244665.5	75907.9	255321.3	611310.8	1040348.5	227856.4	141762.3
MHK	3658319.6	315431.2	3793.6	5701.3	1516.9	26116.5	7677.9	18217.0	12126.6	2301.7
MFA	584.2	415.4	69.4	232.1	334.8	71.0	443.2	219.6	93.5	141.4
MCV	635.4	417.1	56.4	309.1	706.5	32.2	587.1	155.7	432.7	1290.0
MWOHK	6235.5	1905.9	548.0	10191.0	1.9	2050.1	18154.1	95300.3	11497.2	1243.6
MWOFA	22.8	176.7	2.1	5.9	2.3	123.8	88.3	112.7	53.2	41.3
MWOCV	55.4	80.9	11.8	72.1	86.0	14.5	66.5	12.2	51.0	22.1
MW1HK	652189.1	15313.1	783.3	1330.1	26.3	24711.1	3489.4	3612.8	903.8	349.1
MW1FA	5476.1	3642.6	156.8	277.5	614.8	281.3	386.2	354.5	239.0	353.0
MW1CV	2.1	2674.9	534.0	1332.8	11205.7	84.8	258.0	506.3	946.1	2145.7

Table A. 60. MSE result when the magnitude of outlier is 5, $\rho = 0.999$ and $\sigma^2 = 100$.

$\delta^2 = 2$	$\sigma^2 = 100$		$k = 5$							
$\rho = 0.999$			p=3				p=6			
Estimators	15	20	30	50	100	15	20	30	50	100
OLS	6925706.8	12776854.6	563839.6	3969426.0	1271661.1	1855946.7	5682793.7	11982219.1	3417956.8	2438704.0
MHK	278026.3	53249.9	1956.9	5212.3	3419.4	32065.2	45715.4	16887.4	6056.5	2063.3
MFA	366.9	404.1	216.1	463.2	959.2	133.3	422.4	500.7	352.1	611.2
MCV	630.2	1445.4	455.4	3557.3	15.8	71.1	184.6	1597.3	2783.6	6648.9
MWOHK	14814.5	68811.6	15613.9	152545.5	1.7	21184.3	359931.9	2051493.7	295222.5	31102.6
MWOFA	118.3	850.5	2.6	37.4	3.7	678.8	557.1	285.2	262.1	155.6
MWOCV	10.2	249.8	492.5	215.0	238.3	13.3	1.1	11.2	261.4	39.1
MW1HK	8218.2	8851.0	781.8	802.8	2.7	6434.1	3433.5	19381.7	684.6	195.7
MW1FA	292.4	973.0	250.1	429.3	1002.2	146.8	349.6	518.7	385.7	722.5
MW1CV	2413.1	1623.2	386.1	5754.7	62437.7	118.7	1447.1	1075.8	4448.6	15023.2